Errata: Transitive Regret

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The aim of this errata is to correct and clarify the statement of Proposition 1 in [1].

This proposition is the first step in proving the main result, Theorem 1. So it is implicit that the assumptions of Theorem 1 are invoked in proving Proposition 1. We did not include the assumptions of completeness and monotonicity in the statement of this proposition, which may have caused confusion. As pointed out by Chang and Liu [4], Proposition 1 is not true without completeness.

Another issue is that we did not define the notion of continuity and monotonicity that the preference relation satisfies. We rectify this omission and then restate Proposition 1 with all the assumptions. Throughout we use the notation in [1].

Definition 1 A sequence of random variables X^k with the cdf's F^k *converges in distribution* to X with the cdf F, denoted $X^k \xrightarrow{d} X$, if

$$\lim_{k\to\infty} F^k(x) = F(x)$$

at every x at which F is continuous (see Billingsley [3, p. 338]).

Our paper assumes the following notions of continuity and monotonicity of preferences:

Continuity: A preference relation $\succeq$ is *continuous w.r.t. convergence in distribution* if $X^k \succeq Y$ for all k and $X^k \xrightarrow{d} X$ implies $X \succeq Y$ and $Y \succeq X^k$ for all k and $X^k \xrightarrow{d} X$ implies $Y \succeq X$.

State-wise Monotonicity: Let $X = (x_1, S_1; \ldots ; x_n, S_n)$ and $Y = (y_1, S_1; \ldots ; y_n, S_n)$. If for all i, $x_i \geqslant y_i$, then $X \succeq Y$. If in addition for some i $x_i > y_i$, then $X \succ Y$.

Proposition 1 (Probabilistic equivalence). Let $\succeq$ be a complete, transitive, continuous w.r.t. convergence in distribution, and state-wise monotonic, regret-based preference relation over $\mathcal{L}$. For any two random variables $X, Y \in \mathcal{L}$, if $F_X = F_Y$, then $X \sim Y$.

It follows directly from completeness, transitivity, and the continuity assumption that if X and Y have the same distribution, then $X \sim Y$. In other words, the conclusion of Proposition 1 is assumed directly: If $F_X = F_Y$, then $X \sim Y$.

Also observe that since all random variables have a finite number of different outcomes, state-wise monotonicity and Proposition 1 imply:

Monotonicity If X (strictly) dominates Y by first order stochastic dominance, then $X \succeq Y$ ($X \succ Y$).

Proposition 1 can be proved even if convergence in distribution is replaced with convergence in probability (see Billingsley [3, p. 274] for a definition of this type of convergence). For a proof, see [2].

References

[1] Bikhchandani, S. and U. Segal, (2011). "Transitive regret," *Theoretical Economics* 6:95–108.

[2] Bikhchandani, S. and U. Segal, (2024). "Continuity and Monotonicity of Preferences and Probabilistic Equivalence," https://arxiv.org/abs/2409.17529.

[3] Billingsley, P., (1979). *Probability and Measure*. New York: John Wiley & Sons.

[4] Chang, Y. and S. L. Liu (2024). "Counterexamples to 'Transitive Regret,' " https://arxiv.org/abs/2407.00055.