Supplement to "Agendas in legislative decision-making"

(*Theoretical Economics*, Vol. 16, No. 1, January 2021, 235–274)

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1. Connection to Apesteguia, Ballester, and Masatlioglu (2014)

Apesteguia et al. also characterize implementation by Euro–Latin and Anglo–American agenda. Unlike Theorem 2 of the main text, they emphasize the differences between the two formats rather than the similarities.

Condorcet Priority (CP). *For each* $A \in \mathbf{X}$ *such that* $|A| \geq 3$, *there exists a* prioritarian alternative $p^* \in A$ *such that, for every Condorcet triple* P_{p^*xy} *involving* $x, y \in A$, $v(P_{p^*xy}, \{p^*, x, y\}) = p^*$.

Condorcet Anti-Priority (CA). *For each issue* $A \in \mathbf{X}$ *such that* $|A| \geq 3$, *there exists an* anti-prioritarian alternative $p_* \in A$ *such that, for every Condorcet triple* P_{p_*xy} *involving* $x, y \in A$, $v(P_{p_*xy}, \{p_*, x, y\}) = y$.

To characterize implementation by Euro–Latin agenda, they require CP, ILA, and *Division consistency* (DC) (defined in footnote 21 of the main text). For implementation by Anglo–American agenda, they require CA, ILA, and a property called *Elimination consistency* (EC). When the decision rule v satisfies IS and ILA, CP ensures that v marginalizes two alternatives for every issue with three or more alternatives while CA ensures that v marginalizes one alternative.

Claim A. *Suppose that v satisfies IS and ILA. (a) If v satisfies CP, then every every issue A with three or more alternatives has two marginal alternatives. (b) If v satisfies CA, then every issue A with three or more alternatives has a unique marginal alternative (namely the unique anti-prioritarian alternative).*

Proof. The proof of (a) [respectively (b)] is by strong induction on $|A| \equiv m$. For the base case $m = 3$, the claim follows from ILA and CP [respectively CA]. To complete the induction, suppose that the claim holds for $m \leq n$ and consider $m = n + 1$. Where (B, C) is a splitting of A, there are two possibilities for a prioritarian [respectively anti-prioritarian] alternative p in A: (i) $p \in B \cap C$; and, (ii) $p \in B \setminus C$. I address these possibilities separately for (a) and (b).

(a) Consider $b \in B \setminus C$ and $c \in C \setminus B$. By Claim 4(ii) of the main text, (i) leads to the contradiction that p is not prioritarian in $\{b, c, p\}$. So, (ii) must obtain. Using the same

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Available at https://econtheory.org. https://doi.org/10.3982/TE2618

kind of reasoning, it can be shown that $B = \{p\}$. (The idea is to suppose that there exists some $b' \in B \setminus p$. Then, consider an issue $\{b', c, p\}$ such that $c \in C \setminus B$. While there are several cases to consider, a contradiction obtains for each.) By the induction hypothesis, $A \setminus p$ has two marginal alternatives. Since the splitting of A is $(p, A \setminus p)$, IS implies that these alternatives are likewise marginal for A.

(b) Consider $b \in B$ and $c \in C \setminus B$. By Claim 4(ii) of the main text, (ii) leads to the contradiction that p is not anti-prioritarian in $\{b, c, p\}$. So, (i) must obtain. By the induction hypothesis, p is marginal for B and C (since it is anti-prioritarian for these issues). By IS, it then follows that p is marginal for A. Finally, Claim 12 of the main text and the induction hypothesis imply that there can be no other marginal alternative for A. $\square$

Given Theorem 2 of the main text, Claim A implies that one can replace the conditions DC and EC by IS in the characterization results of Apesteguia et al.:

Corollary. *A decision rule v is implementable by:*

(a) Euro–Latin agenda if and only if it satisfies IS, ILA, and CP;

(b) Anglo–American agenda if and only if it satisfies IS, ILA, and CA.

Proof. *(Sufficiency)* By Claim A and Theorem 2 of the main text. *(Necessity)* IS and ILA are necessary by Theorem 2 of the main text. (a) CP is necessary by Theorem 1 of Apesteguia et al. (b) CA is necessary by Theorem 2 of Apesteguia et al. $\square$

2. Marginalization

Note: For ease of presentation, I frequently abuse notation by referring to a node q of a history-independent agenda by its label $\ell(q)$. (Since the agenda is history-independent, this creates no possibility for confusion.)

Knockout agendas (as defined in Section 2 of the main text) can be characterized in terms of marginalization. In particular, they are the only simple agendas where marginalization never "targets" just one alternative.[1]

No targeting. *For every issue $A \in \mathbf{X}$, v marginalizes an alternative only if it marginalizes more than one alternative.*

Theorem I. *A decision rule v is implementable by a knockout agenda if and only if it satisfies Issue splitting, Independence of losing alternatives, and No targeting.*

Proof. *(Sufficiency)* Given IS and ILA, Theorem 1 of the main text implies S^v is a simple agenda implementing v. If S^v is not a knockout agenda, then some alternative $x \in X$

[1]This is not the only way to characterize knockout agendas. Knockout agendas are also the only simple agendas where, for every issue $A \in \mathbf{X}$, the splitting is partitional. Given this observation, it follows that v is implementable by a knockout agenda if and only if it satisfies Division Consistency (see footnote 21 of the main text) and selects the Condorcet winner whenever it exists.

appears at two (or more) terminal nodes. Let A denote a non-terminal node with successors B and C such that $x \in B \cap C$. Since $\mathbb{S}^v$ is simple, it is recursive by Claim 9 of the main text. So, there exist alternatives $b \in B \setminus C$ and $c \in C \setminus B$. By definition, (B, C) splits A. By Claim 4(ii) of the main text, $(\{x, b\}, \{x, c\})$ splits the sub-issue $\{x, b, c\}$. It follows that v marginalizes x alone on $\{x, b, c\}$, which contradicts No targeting. *(Necessity)* Theorem 1 of the main text establishes the necessity of IS and ILA. The necessity of No targeting is obvious. □

Another class of simple agendas that can be characterized in terms of marginalization is the class of standardized simple agendas. Following Miller (1995), an agenda is *standard* if the last question always involves the status quo $\emptyset$ (see footnote 15 of the main text). Given an agenda $\mathbb{T}_A$ (such that $\emptyset \notin A$), the *standardized* agenda $\mathbb{T}_{A\cup\emptyset}$ is obtained by adding, at each terminal node t of $\mathbb{T}_A$, a vote between $\ell(t)$ and $\emptyset$. To illustrate, the agenda in Example 1 of the main text is a standardized Euro–Latin agenda while the agenda in Example 5 is a standardized version of the bill-by-bill agenda in Example 2. (Similarly, the agenda in Figure 14 of Ordeshook and Schwartz (1987) is a standardized bill-by-bill agenda; the agenda in their Figure 5 is a standardized priority agenda; and, the agenda studied by Krehbiel and Rivers is a standardized knockout agenda.)

Since $\emptyset$ appears in every terminal sub-game of a standard agenda, it must be marginal on X (by Lemma 2.2 of Iglesias, Ince, and Loh (2014)).[2] So, implementation by standard agenda imposes a "weak" marginalization requirement:

WEAK MARGINALIZATION. *On the (universal) issue X, v marginalizes some alternative.*

This condition is also satisfied by Euro–Latin agendas. So, it does not distinguish standardized simple agendas from all other simple agendas. To rule out Euro–Latin agendas specifically, one might impose the stronger requirement that v never marginalizes the alternative that is marginal on the universal set X together with another alternative.

WEAK* MARGINALIZATION. *There is some alternative $x_* \in X$ such that: (i) v marginalizes x_* on the issue X; and, (ii) if v marginalizes multiple alternatives on an issue $A \in \mathbf{X}$ with more than two alternatives, then $x_* \notin A$.*

This property is sufficient to distinguish standardized simple agendas from all other simple agendas.

THEOREM II. (Necessity) *Every decision rule v implementable by a standard agenda satisfies Weak marginalization.* (Sufficiency) *What is more, a decision rule v is implementable by a standardized simple agenda if (and only if) it satisfies Issue splitting, Independence of losing alternatives, and Weak* marginalization.*

[2] To see this, fix a profile P where $\emptyset$ is *not* the Condorcet winner. In the first step of the "backward induction" algorithm, an alternative in $X \setminus \emptyset$ can survive if and only if it is majority preferred to $\emptyset$. In each subsequent step of the algorithm, it follows that some alternative which is majority preferred to $\emptyset$ must survive. As a result, $\emptyset$ cannot be the outcome for (P, X).

Proof. *(Sufficiency)* By Weak* marginalization, some alternative x_* is marginal on X. By Claim 12 of the main text, x_* is marginal on all $A \in \mathbf{X}$ such that $x_* \in A$. Now, fix an issue A such that $|A| > 2$ and $x_* \in A$. Let (B, C) denote its splitting. By Claim 14(i) of the main text: (i) $x_* \in B \cap C$ with $b \in B \setminus C$ and $c \in C \setminus B$; or, (ii) $(B, C) = (b, X \setminus b)$ with $x_* \neq b$. By way of contradiction, suppose that (ii) obtains. By Claim 4(ii), $(b, \{x_*, c\})$ splits $D = \{b, c, x_*\}$ for some $c \in A$. So, x_* and c are marginal on D, which contradicts Weak* marginalization. So, $x_* \in B \cap C$ for each $A \in \mathbf{X}$ such that $|A| \geq 3$ and $x_* \in A$. So, $\mathtt{S}^v$ is a standard agenda with x_* as the status quo. Since v satisfies IS and ILA, Theorem 1 of the main text implies that $\mathtt{S}^v$ is simple and implements v. *(Necessity)* The necessity of Weak* marginalization follows from the discussion above. □

3. Non-repetitive and continuous agendas

Claim B. *An agenda is non-repetitive and continuous if and only if, for every non-terminal node q and each successor q_i (with $i = y, n$), there exists an $x_i^q \in c_i(q) \equiv c(q) \cap \ell(q_i)$ that labels exactly one terminal node below q_i.*

Proof. Fix a non-terminal node q of $\mathtt{T}$. $(\Rightarrow)$ Since $\mathtt{T}$ is non-repetitive, $c_i(q) \neq \emptyset$. Since $\mathtt{T}$ is continuous, some $x_i \in c_i(q)$ labels exactly one terminal node below q_i for $i = y, n$. $(\Leftarrow)$ By assumption, $c_i(q) \neq \emptyset$ for $i = y, n$. So, $\mathtt{T}$ is non-repetitive. By assumption, some $x_i \in c_i(q)$ labels exactly one terminal node below q_i for $i = y, n$. So, $\mathtt{T}$ is continuous. □

Claim C. *If an agenda is simple, then it is continuous.*

Proof. Fix a simple agenda $\mathtt{S}$. The proof is by strong induction on $|X| \equiv m$.

The base cases $m = 2, 3$ follow from Claim 9 of the main text. To complete the induction, suppose that the claim holds for $m \leq n$ and consider $m = n + 1$. Let r denote the root. Since $\mathtt{S}$ is simple, the agenda $\mathtt{S}(r_i)$ is simple for $i = y, n$. Since $\mathtt{S}$ is non-repetitive by Claim 9 of the main text, $\ell(r_i) \subset X$ for $i = y, n$. So, each $\mathtt{S}(r_i)$ is continuous by the induction hypothesis. By Claim 9 of the main text, each $\mathtt{S}(r_i)$ is also non-repetitive. Let r_{iy} and r_{in} denote the successors of r_i. By Claim B, some $x_{ij} \in c_j(r_i) \equiv c(r_i) \cap \ell(r_{ij})$ labels exactly one terminal node below r_{ij} for $j = y, n$.

Given Claim B, the proof is complete if $x_{iy} \in c_i(r) \equiv c(r) \cap \ell(r_i)$ or $x_{in} \in c_i(r)$ for $i = y, n$. By way of contradiction, suppose that $x_{iy}, x_{in} \in u(r)$. Now, consider any terminal node t below r_i. By persistence, there exists a node q^t (between r and t) with a successor labeled $u(r)$. Since $x_{iy} \in c_y(r_i)$ and $x_{in} \in c_n(r_i)$, it cannot be any node between r_i and t. So, $q^t = r$ is the only possibility. Hence, $u(r) \in \{\ell(r_y), \ell(r_n)\}$. Without loss of generality, suppose that $\ell(r_y) = u(r)$. Then, $\ell(r_n) = c(r) \cup u(r) = \ell(r)$. But, this contradicts the fact that $\mathtt{S}$ is non-repetitive (by Claim 9 of the main text). □

Remark 7. Every simple agenda is non-repetitive and continuous.

Proof. By Claim C and Claim 9 of the main text. □

References

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Co-editor Dilip Mookherjee handled this manuscript.

Manuscript received 30 August, 2016; final version accepted 11 May, 2020; available online 14 May, 2020.