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Innovation adoption by forward-looking social learners

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We build a model studying the effect of an economy's potential for social learning on the adoption of innovations of uncertain quality. Assuming consumers are forward-looking (i.e., recognize the value of waiting for information), we analyze how qualitative and quantitative features of the learning environment affect equilibrium adoption dynamics, welfare, and the speed of learning. Based on this, we show how differences in the learning environment translate into observable differences in adoption dynamics, suggesting a purely informational channel for two commonly documented adoption patterns: S-shaped and concave curves. We also identify environments that are subject to a saturation effect: Increased opportunities for social learning can slow down adoption and learning, and do not increase consumer welfare, possibly even being harmful.

Keywords. Innovation adoption, social learning, informational free-riding, strategic experimentation, exponential bandits.

JEL classification. D80, D83, O33.

1. Introduction

Suppose a new product of uncertain quality, such as a novel elective medical procedure (e.g., Lasik eye surgery or bariatric weight-loss surgery) or a new movie, is released into the market. In recent years, the rise of online review sites, search engines, video-sharing platforms, and social networking sites has greatly increased the potential for social learning in the economy: If other patients suffer a serious complication or many viewers enjoy the movie, this is more likely than ever to find its way into the public domain; and there are more and more people who have access to this common pool of consumer-generated information.

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This paper builds a model studying the effect of an economy's potential for social learning on the adoption of innovations of uncertain quality. A central ingredient of our model is that consumers are *forward-looking* social learners: In choosing whether to adopt an innovation, they recognize the value of delaying their decision to learn from other adopters' consumption experiences.[1] We analyze how consumers' delay incentives depend on qualitative and quantitative features of the learning environment, and how this affects equilibrium adoption, welfare, and the speed of learning. Our analysis has two main implications. First, qualitatively, we show how differences in the learning environment translate into observable differences in adoption dynamics. This implies a new, purely informational channel for two of the most commonly documented adoption patterns: S-shaped and concave curves. Second, quantitatively, we suggest caution in evaluating the impact of increases in the potential for social learning. We identify environments that are subject to a saturation effect, whereby beyond a certain level, increased opportunities for social learning can slow down adoption and learning, and do not improve consumer welfare (possibly even being harmful).

In our model (Section 2), an innovation of fixed, but uncertain quality (better or worse than the status quo) is introduced to a large population of forward-looking consumers. Consumers are (ex ante) identical, sharing the same prior about the quality of the innovation, the same discount rate, and the same tastes for good and bad quality. At each instant $t \in \mathbb{R}_+$, consumers receive stochastic opportunities to adopt the innovation. A consumer who receives an opportunity must choose whether to irreversibly adopt the innovation or to delay his decision until the next opportunity. In equilibrium, consumers optimally trade off the opportunity cost of delays against the benefit of learning more about the quality of the innovation.

Learning is summarized by a public signal process, representing news that is obtained endogenously—based on the experiences of previous adopters—and possibly also from exogenous sources (e.g., watchdog agencies, professional critics). To study the importance of quantitative and qualitative features of the news environment, we build on the exponential-bandit framework widely used in the literature on strategic experimentation (see Section 1.1): Individual adopters' experiences generate public signals at a fixed Poisson rate that we use to quantify the potential for social learning. Qualitatively, as we interpret in Section 2.2, there is a natural distinction between bad news markets, where signal arrivals (breakdowns) indicate bad quality and the absence of signals makes consumers more optimistic about the innovation, and good news markets, where signals (breakthroughs) suggest good quality and the absence of signals makes consumers more pessimistic.

Section 3 analyzes and contrasts equilibrium adoption dynamics in bad and good news markets. As in many applications of Poisson learning, we focus on the stark but tractable case of *perfect* bad (respectively, good) news, where a single signal arrival conclusively indicates bad (respectively, good) quality. Thus, incentives are nontrivial only absent signals. As a preliminary step, Lemma 1 shows that equilibrium incentives over

[1]Forward-looking social learning is well documented empirically, e.g., in the development economics literature studying the adoption of agricultural innovations (see Section 4.2).

time satisfy a single-crossing property: Absent signals, there is at most one transition from preference for adoption to preference for waiting, or vice versa, with a possible period of indifference in between. Building on this, Theorems 1 and 2 establish equilibrium existence and uniqueness under bad and good news.[2] Equilibrium adoption dynamics admit simple closed-form descriptions that are Markovian in current beliefs and in the mass of consumers who have not yet adopted.

Under bad news, the unique equilibrium is characterized by two cutoff times $0 \leq t_1^* \leq t_2^*$. Until t_1^*, no adoption takes place and consumers acquire information only from exogenous sources; from t_2^* on, all consumers adopt immediately when given a chance (absent breakdowns). If $t_1^* < t_2^*$, then throughout (t_1^*, t_2^*) there is *partial adoption*: Only some consumers adopt when given a chance, with others free-riding on the information generated by the adopters, where the flow of adopters on (t_1^*, t_2^*) ensures indifference between adopting and delaying throughout this interval. A period of partial adoption arises in economies with a large enough potential for social learning, and with sufficiently patient and not too optimistic consumers; otherwise, there is no partial adoption. By contrast, the unique good news equilibrium is always *all-or-nothing*, featuring immediate adoption up to some time t^* and no adoption from t^* on (absent breakthroughs). Thus, regardless of the potential for social learning, consumers' discount rate, or prior beliefs, there is never any partial adoption. This highlights a new distinction between the way in which bad and goods news learning affects consumers' incentives. Specifically, as Section 3.3 explains, sustaining periods of indifference between immediate adoption and delay requires the prospect of receiving news that makes consumers (instantaneously) go from being willing to adopt to being unwilling to adopt; breakdowns have this effect, but breakthroughs do not.

We highlight two implications of our analysis. Section 4.1 shows that, depending on the informational environment, our model generates two commonly documented adoption curves (e.g., Hoyer, MacInnis, and Pieters (2012), Keillor (2007)). Bad news equilibria with $t_1^* < t_2^*$ lead to the leading empirical pattern of S-shaped adoption: Absent breakdowns, the share of adopters increases convexly throughout the partial adoption phase (t_1^*, t_2^*), as convex growth ensures that, despite becoming increasingly optimistic, consumers remain indifferent between adopting and delaying; during the immediate adoption phase from t_2^* on, adoption is concave, reflecting the gradual depletion of the population. In contrast, the all-or-nothing structure of good news equilibria (or bad news equilibria with $t_1^* = t_2^*$) leads to purely concave adoption curves.

Section 4.2 considers increases in the potential for social learning. Proposition 1 establishes a saturation effect: If learning is via bad news and the equilibrium features partial adoption, then such increases are (ex ante) welfare-neutral. Indeed, they are balanced out by an expansion of the period (t_1^*, t_2^*) of informational free-riding, which slows down the adoption of (both good and bad) products and has a non-monotonic effect on the speed of learning. More strongly, with heterogeneous consumers, increased opportunities for social learning can be Pareto-harmful (Remark 1). By contrast, in environments where equilibrium is all-or-nothing, increasing the potential for social learning is (essentially) always strictly beneficial and speeds up learning at all times.

[2]Uniqueness is in terms of aggregate adoption behavior.

1.1 *Related literature*

We study a model of innovation adoption with endogenous timing and social learning from public information. Related informational externalities and strategic delay incentives are analyzed in the literature on observational learning with endogenous timing;[3] see, e.g., Chamley and Gale (1994) and, more closely related, Murto and Välimäki (2011), where players privately obtain Poisson signals about the quality of a risky project at a fixed exogenous rate until they choose to irreversibly exit to a safe outside option. A key difference is that in this literature, players hold private information about the state and draw inferences from others' actions, whereas news in our model is public and derived from previous adopters' experiences. Information aggregates in random bursts in these models rather than smoothly as in our setting, and the aforementioned papers do not derive adoption curves or study how they are shaped by the informational environment.

Our public learning model builds on the framework of strategic experimentation with exponential bandits, originating with Keller, Rady, and Cripps (2005) and Keller and Rady (2010, 2015) (for a survey, see Hörner and Skrzypacz (2017)). We depart in two main ways. First, we study irreversible adoption (i.e., exit to the risky arm), rather than allowing for continuous back-and-forth switching. Second, we assume a continuum of agents, who each have a negligible influence on public information. These departures entail a qualitative difference between bad and good news learning—the presence vs. absence of partial adoption regions—that has observable implications for adoption curves and is absent in the aforementioned papers, where the symmetric Markov equilibrium features a region of partial adoption/mixing under both bad and good news.[4] Another implication of these departures is that, unlike strategic experimentation, our setting does not feature an "encouragement effect," i.e., an incentive to increase current experimentation to drive up beliefs and induce more future experimentation by others. This yields new comparative statics that isolate the impact of informational free-riding: For example, in the bad news environment of Keller and Rady (2015), an increase in the number of players or signal informativeness makes players more willing to experiment at pessimistic beliefs, whereas the saturation effect in Proposition 1 relies on the opposite effect. More recently, Laiho, Murto, and Salmi (2024) study informational free-riding incentives in a related model of collective experimentation with irreversible adoption and a continuum of (heterogeneous) agents, focusing, however, on Brownian news and learning from the stock rather than the flow of adopters.[5]

A large literature in economics, marketing, and sociology seeks to explain why innovations diffuse gradually and why S-shaped (and to a lesser extent concave) adoption

[3]A large literature studies observational learning/innovation adoption with exogenous timing (e.g., Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992), Smith and Sørensen (2000), Herrera and Hörner (2013), Board and Meyer-ter-Vehn (2021)). Strategic delay incentives without social learning are at the center of the literature on wars of attrition (e.g., Maynard Smith (1974), Fudenberg and Tirole (1986), Anderson, Smith, and Park (2017)).

[4]Bonatti and Hörner (2017) study a different departure—unobservable actions—and find that this also leads to the symmetric equilibrium under bad vs. good news being in mixed vs. pure strategies.

[5]Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017) study strategic investment timing by a continuum of agents whose investment produces Gaussian public signals about an evolving state. They show that this generates self-reinforcing episodes of high uncertainty and low investment.

patterns are prevalent. We relate to other learning-based models of these phenomena.[6] Unlike existing work that focuses on social learning by myopic consumers (e.g., Young (2009)) or forward-looking learning from exogenous signals (e.g., Jensen (1982)), we consider a model of forward-looking social learning. This allows us to provide a purely informational explanation of observed adoption patterns, whereas models with myopic consumers or exogenous signals require specific forms of consumer heterogeneity to generate S-shaped adoption.[7] Our saturation effect also hinges on the combination of forward-looking incentives and social learning, as under myopic or exogenous learning, a greater ease of information transmission is always beneficial.

Our focus on the informational determinants of innovation adoption contrasts with work that combines informational and payoff externalities. Rob (1991) models entry into a new market, where the current number of firms in the market influences not only entrants' learning about a demand parameter, but also their profits via the market price. Related to our bad news equilibrium, equilibrium entry is pinned down by a zero profits condition and is lower than socially optimal. He does not study how the informational environment affects entry dynamics or provide conditions for S-shaped growth, both of which would also depend on the inverse demand function. Bergemann and Välimäki (1997) obtain S-shaped adoption as a result of duopolistic competition between an established and a new seller in a model with reversible adoption and learning on both the buyer and the seller side. Initial adoption of the new product exceeds the social optimum in their model. Laiho and Salmi (2018) build on our model by incorporating monopoly pricing and consumer heterogeneity.

2. Model

2.1 *The game*

Time $t \in \mathbb{R}_+$ is continuous. At time $t = 0$, an innovation of unknown quality $\theta \in \{G = 1, B = -1\}$ and of unlimited supply is released to a continuum population of potential consumers of mass $N_0 \in \mathbb{R}_{>0}$. Consumers are ex ante identical. They have a common prior $p_0 \in (0, 1)$ that $\theta = G$, they are forward-looking with common discount rate $r > 0$, and they have the same actions and payoffs, as specified below.

At each time t, consumers receive stochastic opportunities to adopt the innovation. Adoption opportunities are generated independently across consumers and histories according to a Poisson process with exogenous arrival rate $\rho > 0$.[8] Given an adoption

[6] Non-informational models (for surveys, see Baptista (1999), Geroski (2000)) include "epidemic" models (e.g., Mansfield (1961), Bass (1969)), "probit" models of heterogeneously evolving benefits to adoption (e.g., Davies (1979)), and models of pure payoff externalities (e.g., Jovanovic and Lach (1989), Farrell and Saloner (1986)). Wolitzky (2018) contrasts adoption levels of cost-saving vs. outcome-improving innovations in a model of learning from others' outcomes. Che and Hörner (2018) take a mechanism design approach to incentivizing social learning about an innovation.

[7] In those models, agents adopt if and only if their beliefs exceed a cutoff. This precludes regions of convex adoption with identical agents, instead requiring specific distributions of heterogeneous priors/tastes.

[8] In the context of our motivating examples, stochastic adoption opportunities may represent, e.g., convenient times to take off work to undergo an elective surgery or a free evening to watch a movie. Section 5 discusses the case when $\rho \to \infty$.

opportunity, a consumer must choose whether to adopt the innovation ($a_t = 1$) or to wait ($a_t = 0$). If a consumer adopts, he receives an expected lump sum payoff of $\mathbb{E}_t[\theta]$, conditioned on information available up to time t, and drops out of the game.[9] If the consumer chooses to wait or does not receive an adoption opportunity at t, he receives a flow payoff of 0 until his next adoption opportunity, where he faces the same decision again.

2.2 *Learning*

Over time, consumers observe public signals that convey information about the quality of the innovation. We employ a variation of the Poisson learning models used in the literature on strategic experimentation. Let n_t denote the flow of of consumers newly adopting the innovation at time t, which we define more precisely in Section 2.3. Conditional on the quality of the innovation being θ, public signals arrive according to an inhomogeneous Poisson process with arrival rate $\varepsilon_\theta + \lambda_\theta n_t$, where $\lambda_\theta > 0$ and $\varepsilon_\theta \geq 0$ are exogenous parameters that depend on θ.

The signal process summarizes news events that are generated from two sources. First, the *social learning* term λn_t represents news generated endogenously, based on the experiences of other consumers. It captures a flow n_t of new adopters each generating signals at rate λ.[10] Thus, the greater the flow of consumers adopting the innovation at t, the more likely it is for a signal to arrive at t; hence, the absence of a signal at t is more informative the larger is n_t. Second, we also allow for (but do not require) signals to arrive at a fixed *exogenous* rate ε, representing information generated independently of consumers' behavior (e.g., by watchdog agencies or professional critics).

As in many applications of Poisson learning, we focus for tractability on perfect news processes, where a single signal provides conclusive evidence of the quality of the innovation. Qualitatively, there is a natural distinction between two types of news environments. Learning is via *perfect bad news* (for short, *bad news*) if $\varepsilon_G = \lambda_G = 0$ and $\varepsilon_B = \varepsilon \geq 0$, $\lambda_B = \lambda > 0$; that is, the arrival of a signal (a *breakdown*) is conclusive evidence that the innovation is bad. Learning is via *perfect good news* (for short, *good news*) if $\varepsilon_B = \lambda_B = 0$ and $\varepsilon_G = \varepsilon \geq 0$, $\lambda_G = \lambda > 0$; that is, a signal arrival (a *breakthrough*) is conclusive evidence that the innovation is good. The nature of the news environment may be influenced by whether a bad or good quality innovation is more likely to generate newsworthy (e.g., extreme) payoff realizations. For example, an unsafe medical procedure may cause serious complications that are widely reported, but a safe procedure that performs as intended may not lead to newsworthy outcomes.[11] Alternatively, the

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[9] Irreversible adoption is natural for innovations such as medical procedures or movies, for which "consumption" is typically a one-time event, or for technologies with large switching costs.

[10] We obtain qualitatively similar results when the social learning component at time t is taken to depend on the stock, $\int_0^t n_s\,ds$, rather than the flow of adopters at t. See Section 5.

[11] More generally, suppose payoffs of the quality θ innovation are drawn (independently across consumers) from cumulative distribution function F_θ, where $\int_{-\infty}^{\infty} \xi\, dF_\theta(\xi) = \theta$. Suppose payoff realizations ξ are newsworthy if and only if $\xi \leq \underline{\xi}$ or $\xi \geq \overline{\xi}$ for some "extreme" low and high payoffs $\underline{\xi} < \overline{\xi}$, and that newsworthy payoffs generate public signals at some rate. Bad news learning assumes $F_B(\underline{\xi}) > 0 = F_G(\underline{\xi})$

news environment may reflect reporting practices of the available social learning systems. For example, several movie review aggregator and streaming sites provide "best of" lists of new releases with the highest user ratings, but do not display "worst of" lists.

Quantitatively, we use $\Lambda_0 := \lambda N_0$ as a simple measure of the *potential for social learning* in the economy, summarizing both the likelihood λ with which individual adopters' experiences find their way into the public domain and the size N_0 of the population that can contribute to and access the common pool of information.

Under bad news, consumers' posterior on $\theta = G$ permanently jumps to 0 at the first breakdown, while under good news, consumers' posterior on $\theta = G$ permanently jumps to 1 at the first breakthrough. Let p_t denote consumers' *no-news posterior*, i.e., the belief at t that $\theta = G$ conditional on no signals having arrived on $[0, t)$. Given a flow of adopters (n_t), Bayesian updating implies[12]

$$p_t = \begin{cases} \dfrac{p_0}{p_0 + (1-p_0)e^{-\int_0^t (\varepsilon+\lambda n_s)\,ds}} & \text{under bad news} \\ \dfrac{p_0 e^{-\int_0^t (\varepsilon+\lambda n_s)\,ds}}{p_0 e^{-\int_0^t (\varepsilon+\lambda n_s)\,ds} + (1-p_0)} & \text{under good news.} \end{cases} \tag{1}$$

In particular, if (n_t) is continuous in t on some open interval, then on this interval (p_t) evolves according to the ordinary differential equation (ODE)

$$\dot{p}_t = \begin{cases} (\varepsilon + \lambda n_t) p_t (1-p_t) & \text{under bad news} \\ -(\varepsilon + \lambda n_t) p_t (1-p_t) & \text{under good news.} \end{cases}$$

Note that the no-news posterior is continuous. Moreover, it is increasing under bad news and decreasing under good news.

2.3 *Equilibrium*

Our interest is in the aggregate adoption dynamics of the population. Thus, our equilibrium concept takes as its primitive the aggregate flow (n_t) of new adopters and does not explicitly model individual consumers' behavior. Given our focus on perfect news processes, incentives are nontrivial only in the absence of signals: Under bad news, no consumers adopt after a breakdown, while under good news, all remaining consumers adopt at their first opportunity after a breakthrough. We henceforth denote by n_t the flow of new adopters at t *conditional on no signals up to time* t and we define equilibrium in terms of this quantity.

Capturing that aggregate adoption is predictable with respect to the public news process, we require (n_t) to be a deterministic function of time. We consider all such functions that are feasible; that is, (n_t) is right-continuous in t and $n_t \in [0, \rho N_t]$ for all $t \in \mathbb{R}_+$, where $N_t := N_0 - \int_0^t n_s\,ds$ denotes the mass of consumers remaining in the game

and $F_B(\overline{\xi}) = F_G(\overline{\xi}) = 1$, i.e., bad innovations sometimes generate extreme low payoffs, but neither good nor bad innovations generate extreme high payoffs. Good news learning assumes $F_B(\underline{\xi}) = F_G(\underline{\xi}) = 0$ and $F_B(\overline{\xi}) = 1 > F_G(\overline{\xi})$.

[12]Section 2.3 imposes measurability on (n_t), so the expressions in (1) are well defined.

at time t. Imposing $n_t \leq \rho N_t$ ensures that at each t, n_t is consistent with all remaining N_t consumers independently receiving adoption opportunities at rate ρ. Any feasible adoption flow (n_t) induces a no-news posterior (p_t) via (1).

In equilibrium, we require that at each t, n_t is consistent with optimal behavior by the remaining N_t forward-looking consumers: Consumers who receive an adoption opportunity at t consider the *expected payoff to adopting* immediately, which is $u_t := 2p_t - 1$ absent news, and optimally trade this off against the value to waiting, taking into account that future adoption evolves according to process (n_t).

Formally, define the *value to waiting* (W_t) associated with process (n_t) to be the solution to the following Bellman equation at each t.[13] Under bad news,

$$W_t = \int_t^\infty \rho e^{-(r+\rho)(s-t)} \underbrace{\left(p_t + (1-p_t)e^{-\int_t^s (\varepsilon+\lambda n_k)\,dk}\right)}_{\substack{\text{prob. of no breakdown} \\ \text{in } [t,s)}} \max\{u_s, W_s\}\,ds;$$

that is, W_t is the expected discounted payoff to waiting until the next stochastic adoption opportunity s, and then adopting at this opportunity if and only if (i) there has been no breakdown and (ii) at the updated belief p_s, the expected payoff to adopting u_s exceeds the new value to waiting W_s.

Under good news,

$$W_t = \int_t^\infty \rho e^{-(r+\rho)(s-t)} \Big(\underbrace{\left(1 - p_t + p_t e^{-\int_t^s (\varepsilon+\lambda n_k)\,dk}\right)}_{\substack{\text{prob. of no breakthrough} \\ \text{in } [t,s)}} \max\{u_s, W_s\} + \underbrace{p_t\left(1 - e^{-\int_t^s (\varepsilon+\lambda n_k)\,dk}\right)}_{\substack{\text{prob. of breakthrough} \\ \text{in } [t,s)}} \Big)\,ds;$$

that is, W_t is the expected discounted payoff to waiting until the next adoption opportunity s, and adopting at this opportunity if either (i) there has been no breakthrough and, at the updated belief p_s, the expected payoff to adopting u_s exceeds the new value to waiting W_s, or (ii) there has been a breakthrough.

Definition 1. An *equilibrium* is a feasible adoption flow (n_t) such that

(i) $W_t \geq u_t$ for all t such that $n_t < \rho N_t$

(ii) $W_t \leq u_t$ for all t such that $0 < n_t$.

Condition (i) says that if some consumers who receive an adoption opportunity at t decide not to adopt, then the value to waiting W_t must weakly exceed the expected payoff to immediate adoption u_t. Similarly, (ii) requires that if some consumers adopt at time t, then the value to waiting must be weakly less than the payoff to immediate

[13] A unique solution exists by standard arguments (e.g., Theorem 3.3 in Stokey, Lucas, and Prescott (1989)).

adoption. Thus, at all times, n_t is consistent with consumers optimally trading off the expected payoff to immediate adoption against the value to waiting.[14]

3. Equilibrium analysis

3.1 *Single-crossing property for equilibrium incentives*

We now proceed to the equilibrium analysis. As a preliminary step, we establish a useful structural property of equilibrium incentives under both bad and good news. Suppose that (n_t) is an arbitrary feasible flow of adopters, with associated no-news posterior (p_t) and value to waiting (W_t). In general, the dynamics of the trade-off between immediate adoption at time t and delaying and behaving optimally in the future can be difficult to characterize, with $u_t - W_t$ changing sign many times. However, when (n_t) is an equilibrium flow, then for any t,

$$[W_t > u_t \Longrightarrow n_t = 0] \quad \text{and} \quad [W_t < u_t \Longrightarrow n_t = \rho N_t],$$

which imposes considerable discipline on the dynamics of the trade-off. Indeed, the following result shows that (u_t) and (W_t) satisfy a single-crossing property: There can be at most one transition from strict preference for adoption to strict preference for waiting, or vice versa, with a possible period of indifference in between.

Lemma 1. *Let (n_t) be an equilibrium with associated no-news payoffs to immediate adoption (u_t) and value to waiting (W_t). Under bad news learning, we have*

$$[W_t < u_t \Rightarrow W_\tau < u_\tau \, \forall \tau > t] \quad \textit{and} \quad [W_t \leq u_t \Rightarrow W_\tau \leq u_\tau \, \forall \tau > t].$$

Under good news learning, we have

$$[W_t > u_t \Rightarrow W_\tau > u_\tau \, \forall \tau > t] \quad \textit{and} \quad [W_t \geq u_t \Rightarrow W_\tau \geq u_\tau \, \forall \tau > t].$$

The proof is provided in Appendix A.2. We briefly illustrate the intuition for the first implication when learning is via bad news. Suppose that immediate adoption is strictly better than waiting today and, hence (by continuity of u_t and W_t), also in the near future provided there are no breakdowns. Then, in equilibrium, in the near future, all consumers adopt upon their first opportunity, so the no-news posterior strictly increases while the mass of remaining consumers (and, hence, the flow of new adopters) strictly decreases. Thus, in the near future, the flow of information decreases over time, as the signal arrival rate is proportional to the flow of new adopters. As a result, immediate adoption becomes even more attractive relative to waiting and, hence, remains strictly preferable at all times in the future.

[14] Definition 1 is essentially Nash equilibrium, i.e., does not impose subgame perfection. The motivation is that in a continuum population, individual consumers' behavior has a negligible impact on aggregate adoption, so any off-path history where the flow of adopters differs from the equilibrium flow is more than a unilateral deviation from the equilibrium path. Thus, off-path histories do not affect individual consumers' incentives on path and are unimportant for equilibrium analysis.

3.2 *Bad news equilibrium*

In this section, we consider learning via bad news. Building on Lemma 1, the following theorem establishes the existence and uniqueness of equilibrium.

THEOREM 1 (Bad News Equilibrium). *Fix $r, \rho, \lambda, N_0 > 0$, $\varepsilon \geq 0$, and $p_0 \in (0, 1)$. There exists a unique equilibrium (n_t). The equilibrium is described by two unique cutoff times $0 \leq t_1^* \leq t_2^* \leq \infty$ such that, absent breakdowns,*

$$n_t = \begin{cases} 0 & \text{if } t \leq t_1^* \\ \dfrac{r(2p_t - 1)}{\lambda(1 - p_t)} - \dfrac{\varepsilon}{\lambda} \in (0, \rho N_t) & \text{if } t \in (t_1^*, t_2^*) \\ \rho N_t & \text{if } t \geq t_2^*. \end{cases} \tag{2}$$

For any bad news equilibrium (n_t), Lemma 1 yields cutoff times $0 \leq t_1^* \leq t_2^* \leq \infty$ such that, absent breakdowns, $n_t = 0$ at all $t < t_1^*$, $n_t = \rho N_t$ at all $t > t_2^*$, and consumers are indifferent between immediate adoption and waiting at all $t \in (t_1^*, t_2^*)$.[15] The proof of Theorem 1 (Appendix A.3) shows that the cutoff times t_1^* and t_2^* are uniquely pinned down by the parameters. Moreover, if $t_1^* < t_2^*$ (as we will see is the case for suitable parameter values), the flow of adopters throughout (t_1^*, t_2^*) is uniquely pinned down as in (2). Below we sketch the argument.

Partial adoption during (t_1^*, t_2^*). Lemma A.6 shows that the flow of adopters at all times $t \in (t_1^*, t_2^*)$ must satisfy $n_t = \frac{r(2p_t-1)}{\lambda(1-p_t)} - \frac{\varepsilon}{\lambda} \in (0, \rho N_t)$. Thus, throughout (t_1^*, t_2^*) there is *partial adoption*: Some consumers adopt when given a chance, while others free-ride on the information generated by the adopters. To illustrate, note that indifference requires the expected payoff from waiting vs. adopting today to be equal, i.e., $W_t - u_t = 0$ for all $t \in (t_1^*, t_2^*)$. Heuristically, by conditioning on the two events that can occur between t and $t + dt$, we can decompose the difference $W_t - u_t$ as

$$\underbrace{(1 - p_t)(\varepsilon + \lambda n_t)\, dt}_{\substack{\text{probability of} \\ \text{breakdown}}} \times \underbrace{1}_{\substack{\text{benefit of waiting:} \\ \text{avoid bad product}}} - \underbrace{\big(1 - (1 - p_t)(\varepsilon + \lambda n_t)\, dt\big)}_{\substack{\text{probability of} \\ \text{no breakdown}}} \underbrace{u_{t+dt} r\, dt}_{\substack{\text{cost of waiting:} \\ \text{discounting}}} = 0. \tag{3}$$

The first term considers the event that a breakdown occurs between t and $t + dt$, which has instantaneous probability $(1 - p_t)(\varepsilon + \lambda n_t)\, dt$. Conditional on this event, the product is bad, so adopting at t yields a payoff of -1, whereas waiting allows consumers to avoid adopting the product, yielding a benefit of $0 - (-1) = 1$. The second term considers the complementary event that no breakdown occurs between t and $t + dt$. Conditional on this event, the probability of good quality is p_{t+dt}, so the expected quality is $2p_{t+dt} - 1 = u_{t+dt}$. Adopting at t yields this payoff immediately. In contrast, the expected

[15]Specifically, let $t_1^* := \inf\{t \geq 0 : n_t > 0\}$ and $t_2^* := \sup\{t \geq 0 : n_t < \rho N_t\}$, with the conventions $\inf \emptyset := \infty$ and $\sup \emptyset := 0$. To see indifference on (t_1^*, t_2^*), note that for any $t \in (t_1^*, t_2^*)$, there exist $k \in [t_1^*, t)$ and $\ell \in (t, t_2^*]$ with $n_k > 0$ and $n_\ell < \rho N_\ell$. Since (n_s) is an equilibrium, this implies $W_k \leq u_k$ and $W_\ell \geq u_\ell$, so $u_t = W_t$ by Lemma 1.

value to waiting is $u_{t+dt}(1 - r\,dt)$: If there is no breakdown between t and $t + dt$, consumers remain indifferent at $t + dt$; hence, regardless of whether or not a consumer receives an adoption opportunity at $t + dt$, his continuation value at $t + dt$ is $W_{t+dt} = u_{t+dt}$, which from the point of view of time t yields $u_{t+dt}(1 - r\,dt)$. Thus, due to discounting, waiting incurs a cost of $u_{t+dt} r\,dt$. Note that ignoring terms of order dt^2, the second term in (3) simplifies to $r(2p_t - 1)\,dt$.[16] Hence, the left-hand side of (3) is increasing in n_t (reflecting that agents' decisions to wait are strategic substitutes), and the cost and benefit of waiting are equalized at the interior adoption flow $n_t = \frac{r(2p_t-1)}{\lambda(1-p_t)} - \frac{\varepsilon}{\lambda}$.[17]

Unique equilibrium. Next, we show how the cutoff times t_1^* and t_2^*, and, hence, the equilibrium flow of adopters (n_t), are uniquely pinned down from the parameters. To illustrate, the following condition rules out values of p_0, ε, and ρ under which (as the Appendix shows) equilibrium adoption is either identically zero (i.e., $t_1^* = t_2^* = \infty$) or there is never partial adoption (i.e., $t_1^* = t_2^*$ regardless of other parameters).

Condition 1. *Suppose (i) either* $\varepsilon > 0$ *or* $p_0 \in (\frac{1}{2}, 1)$ *and (ii)* $\varepsilon < \rho$.

Let p^s denote the cutoff posterior above which adoption occurs in the *single-agent* benchmark, that is, when adoption opportunities arrive at rate ρ and information arrives solely through the exogenous news source at rate ε. This is given by $p^s = \frac{(\varepsilon+r)(r+\rho)}{2(\varepsilon+r)(r+\rho)-\varepsilon\rho}$ (see Appendix A.3). At any belief $p \leq p^s$, every consumer prefers to wait regardless of the flow (n_t) of adopters, as waiting is optimal even when news arrives at the minimal rate ε at all future times. Conversely, at any belief $p \geq p^\sharp := \frac{\rho+r}{\rho+2r} = \lim_{\varepsilon\to\infty} p^s$, every consumer prefers to adopt immediately regardless of (n_t), as immediate adoption is optimal even when news arrives at the maximal rate $\varepsilon \to \infty$. For all intermediate beliefs $p \in (p^s, p^\sharp)$, we show there is a unique critical mass $N^*(p) \in \mathbb{R}_+$ of remaining consumers with the property that if *all* these remaining consumers adopt at their first future opportunity, then a consumer with posterior p is indifferent between immediate adoption and adopting at his next opportunity absent breakdowns. See Figure 1, where we let $N^*(p) = 0$ if $p \leq p^s$ and $N^*(p) = \infty$ if $p \geq p^\sharp$.

Given this, Lemma A.7 characterizes the cutoff times t_1^* and t_2^*. Time t_2^* is the first time at which the remaining mass of consumers N_t drops below the critical mass $N^*(p_t)$ needed to ensure willingness to delay at posterior p_t. Time t_1^* is the first time at which the no-news posterior p_t exceeds $\overline{p} := \frac{\varepsilon+r}{\varepsilon+2r}$ if this occurs before t_2^*; otherwise, $t_1^* = t_2^*$. Indeed, $\overline{p} = \lim_{\rho\to\infty} p^s$ is the belief at which a single agent is indifferent between immediate adoption and delay if adoption opportunities arrive continuously; this is precisely what is needed at the start of the indifference region, as here the value to waiting is independent of ρ (recall footnote 17) and at t_1^*, information arrives solely through the exogenous news source.

As Figure 1 illustrates, this means that equilibrium adoption n_t is Markovian in the no-news posterior p_t and the remaining mass of consumers N_t. Region I, where $N_t \leq$

[16] More precisely, this term is given by $(1 - (1 - p_t)(\varepsilon + \lambda n_t)\,dt)(u_t + \dot{u}_t\,dt)r\,dt = ru_t\,dt = r(2p_t - 1)\,dt$.

[17] Observe that ρ does not enter this expression. This is because, as noted, indifference throughout (t_1^*, t_2^*) implies that consumers' value to waiting at any $t \in (t_1^*, t_2^*)$ does not depend on the arrival rate of their next adoption opportunity.

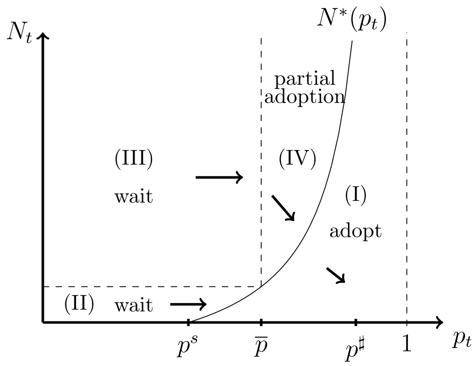

FIGURE 1. Equilibrium dynamics of (p_t, N_t) when $\varepsilon < \rho$.

$N^*(p_t)$, corresponds to immediate adoption (i.e., $n_t = \rho N_t$). Regions II and III, where $p_t < \overline{p}$ and $N_t > N^*(p_t)$, correspond to no adoption (i.e., $n_t = 0$). Region IV, where $p_t \geq \overline{p}$ and $N_t > N^*(p_t)$, corresponds to partial adoption (i.e., $n_t = \frac{r(2p_t - 1)}{\lambda(1-p_t)} - \frac{\varepsilon}{\lambda}$).

From this, it is easy to see how the initial parameters (p_0, N_0) pin down (n_t) (the Appendix verifies feasibility). If (p_0, N_0) starts off in region I, then $t_1^* = t_2^* = 0$, so all consumers adopt at their first opportunity. If (p_0, N_0) is in region IV, then $0 = t_1^* < t_2^*$, so there is an initial period of partial adoption according to the second line of (2), and t_2^* is the first time at which (p_t, N_t) enters region I. Finally, if (p_0, N_0) is in region II (resp. III), then initially everyone delays and p_t drifts up according to $\dot{p}_t = p_t(1 - p_t)\varepsilon$ while N_t remains at N_0; this yields a unique time t_1^* at which (p_t, N_0) enters region I (resp. IV), whence n_t evolves as in the two previous cases.

Conditions for partial adoption. Whether partial adoption arises in equilibrium (i.e., $t_1^* < t_2^*$) depends on the fundamentals. Provided consumers are not too optimistic or impatient (i.e., $p_0 < p^\sharp$),[18] there is a partial adoption phase if and only if the potential for social learning $\Lambda_0 = \lambda N_0$ is large enough.

LEMMA 2. *Fix ρ, ε and p_0 satisfying Condition 1, and $r > 0$. Assume $p_0 < p^\sharp$. Then there exists $\bar{\Lambda}_0 > 0$ such that $t_1^*(\Lambda_0) < t_2^*(\Lambda_0)$ if and only if $\Lambda_0 > \bar{\Lambda}_0$.*[19]

The role of a large enough population N_0 can be seen in Figure 1. If (p_0, N_0) is in region I (i.e., $N_0 < N^*(p_0)$), everyone adopts immediately, as even the maximal rate $\lambda \rho N_0 \, dt$ at which other consumers might generate information is too small to justify delay at belief p_0. Likewise, in region II (i.e., $N_0 \in (N^*(p_0), N^*(\overline{p}))$), all consumers initially delay and learn from exogenous news, but as soon as they become willing to adopt, (p_t, N_0) enters region I, so the information generated by other adopters is again too small to make anyone willing to delay.[20] A too low signal arrival rate λ has the same

[18]Note that $p^\sharp$ is decreasing in the discount rate r.

[19]By the Markovian description above, Λ_0 pins down t_1^* and t_2^* fixing other parameters, because it is sufficient to determine the evolution of p_t and λN_t, which in turn determines t_1^* and t_2^*.

[20]As Section 5 discusses, regions I and II vanish as $\rho \to \infty$, in which case the equilibrium features partial adoption for any $\Lambda_0 > 0$.

effect, as the rate of social learning depends only on λn_t. In particular, if learning is purely exogenous ($\lambda = 0 < \varepsilon$), there is never partial adoption, regardless of other parameters. The same is true if consumers are myopic ($r = \infty$); there, $p^s = p^\sharp = \overline{p} = \frac{1}{2}$, so $t_1^* = t_2^* = \inf\{t : p_t > \frac{1}{2}\}$. Thus, the possibility of partial adoption hinges on the combination of social learning and forward-looking incentives.

3.3 *Good news equilibrium*

Next, consider learning via good news. As under bad news, there is a unique equilibrium. However, regardless of the potential for social learning in the economy, the equilibrium is *all-or-nothing*: There is a time t^* before which all consumers adopt if given an opportunity and after which no consumers adopt absent breakthroughs.

Theorem 2 (Good News Equilibrium). *Fix* $r, \rho, \lambda, N_0 > 0$, $\varepsilon \geq 0$, *and* $p_0 \in (0, 1)$. *There exists a unique equilibrium* (n_t). *The equilibrium is described by a unique cutoff time* $0 \leq t^* \leq \infty$ *such that, absent breakthroughs,*

$$n_t = \begin{cases} \rho N_t & \text{if } t < t^* \\ 0 & \text{if } t \geq t^*. \end{cases} \tag{4}$$

We prove Theorem 2 in Appendix A.5. Similar to the bad news case, in any good news equilibrium (n_t), Lemma 1 yields cutoff times $0 \leq t_1^* \leq t_2^* \leq \infty$, which now have the property that absent breakthroughs, $n_t = \rho N_t$ at all $t < t_1^*$, $n_t = 0$ at all $t > t_2^*$, and consumers are indifferent between immediate adoption and waiting on (t_1^*, t_2^*).[21]

The key difference with bad news is that we must always have $t_1^* = t_2^* =: t^*$, i.e., there cannot be a partial adoption region (Lemma A.9). Note that at any $t < t_2^*$, consumers weakly prefer immediate adoption to waiting, i.e., $W_t \leq u_t$. We show that in fact $W_t < u_t$, so consumers strictly prefer immediate adoption. Heuristically, we can obtain the following upper bound on $W_t - u_t$ by conditioning, similar to (3) under bad news, on the two events that can occur between t and $t + dt$:

$$W_t - u_t \leq \underbrace{p_t(\varepsilon + \lambda n_t)\, dt}_{\substack{\text{probability of}\\ \text{breakthrough}}} \underbrace{\left(\frac{\rho}{r+\rho} - 1\right)}_{\substack{\text{cost of waiting:}\\ \text{delayed adoption}}} - \underbrace{\big(1 - p_t(\varepsilon + \lambda n_t)\, dt\big)}_{\substack{\text{probability of}\\ \text{no breakthrough}}} \underbrace{u_{t+dt} r\, dt}_{\substack{\text{cost of waiting:}\\ \text{discounting}}} < 0. \tag{5}$$

The first term considers the event that a breakthrough occurs between t and $t + dt$, which has instantaneous probability $p_t(\varepsilon + \lambda n_t)\, dt$. Conditional on this event, the product is good, so unlike the case of a breakdown, there is now a cost of $\frac{\rho}{r+\rho} - 1$ to waiting vs. adopting at t, as waiting delays receiving the payoff of 1 until the next adoption opportunity. The second term considers the complementary event that no breakthrough occurs between t and $t + dt$. In this case, there is again a cost to waiting vs. adopting

[21]Specifically, let $t_1^* := \inf\{t \geq 0 : n_t < \rho N_t\}$ and $t_2^* := \sup\{t \geq 0 : n_t > 0\}$. Then indifference on (t_1^*, t_2^*) follows from Lemma 1 by the same logic as under bad news (footnote 15).

at t, which is *at least* $u_{t+dt} r\,dt$. Indeed, conditional on this event, the expected quality is $2p_{t+dt} - 1 = u_{t+dt}$, which is received immediately if the consumer adopts at t. In contrast, the payoff to waiting is at most $u_{t+dt}(1 - r\,dt)$: At $t + dt < t_2^*$, consumers still weakly prefer to adopt, so (regardless of whether or not an adoption opportunity arrives at $t + dt$) the continuation payoff at $t + dt$ is at most u_{t+dt}.[22] which yields $u_{t+dt}(1 - r\,dt)$ from the point of view of time t. Thus, in either event, adopting immediately at t is strictly better than waiting.[23]

The impossibility of a partial adoption region under good news reflects the following fundamental difference with bad news learning. Suppose a consumer is willing to adopt at both t and $t + dt$ absent news. Then he can be willing to delay at t only if the arrival of news between t and $t + dt$ is *decision-relevant*, i.e., would make him strictly prefer *not* to adopt. Indeed, if he anticipates remaining willing to adopt no matter what happens between t and $t + dt$, then (by discounting) he is better off adopting immediately at t.[24] Under bad news, a breakdown between t and $t + dt$ indeed makes the consumer strictly prefer not to adopt. In contrast, under good news, the effect of a breakthrough is to make the consumer strictly prefer to adopt, which is not decision-relevant when he is already willing to adopt.

Finally, to complete the proof of Theorem 2, Lemma A.10 shows that t^* is the unique time at which the no-news posterior p_t reaches the cutoff belief $p^s = \frac{(\varepsilon+r)(r+\rho)}{2(\varepsilon+r)(r+\rho)-\varepsilon\rho}$. Observe that p^s is the same cutoff as in the single-agent benchmark where information is generated solely at the exogenous rate ε (reflecting the absence of an encouragement effect in our setting, as noted in Section 1.1). Thus, consumers' behavior as a function of their current belief does not depend on λ or N_0. Social learning only affects the time t^* at which adoption ceases conditional on no breakthroughs.

4. Implications

We now study the implications of the preceding analysis for observed adoption patterns and for the effect of increased social learning opportunities on welfare, learning, and adoption dynamics.

4.1 *Adoption curves: S-shaped versus concave*

Consider the *adoption curve* of the innovation, which plots the share of adopters in the population against time. Conditional on no news up to time t, this is given by $A_t := \int_0^t n_s/N_0\,ds$. Theorems 1 and 2 yield the following predictions for the shape of A_t.

[22]If an adoption opportunity arrives, the continuation payoff is u_{t+dt}; if not, it is $W_{t+dt} \le u_{t+dt}$.

[23]It is straightforward to verify that $n_t > 0$ and $p_t > \frac{1}{2}$ for all $t < t_2^*$ (see the formal argument in Lemma A.9), so both terms in (5) are *strictly* negative.

[24]If the consumer weakly prefers to adopt at $t + dt$ no matter what, then his continuation payoff to waiting at t is bounded above by the time-t expected discounted payoff to adopting at $t + dt$ (in case of an adoption opportunity at $t + dt$, the continuation value is exactly this; otherwise, it is weakly lower), which by the martingale property of beliefs is $(1 - r\,dt)u_t < u_t$. (The only exception is if $u_t = 0$, but this cannot happen in the interior of a region of positive adoption.).

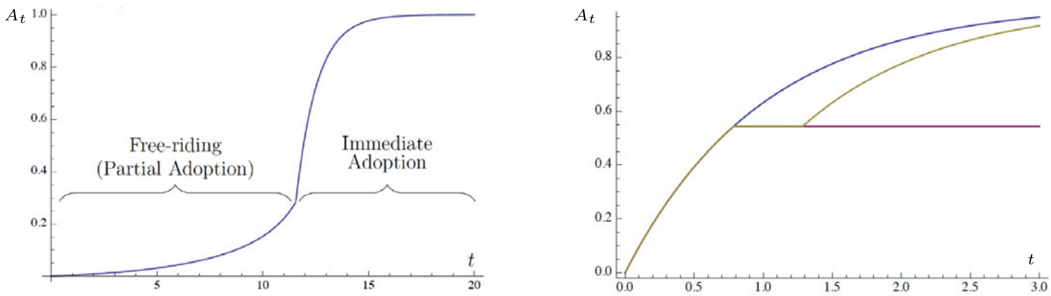

FIGURE 2. Left: The S-shaped adoption curve under bad news conditional on no breakdowns ($t_1^* = 0$). Right: Concave adoption curves under good news (blue = breakthrough before t^*; yellow = breakthrough after t^*; pink = bad quality).

COROLLARY 1. ***Bad News.*** *In the unique equilibrium of Theorem 1, $A_t = 0$ for $0 \le t < t_1^*$, A_t is strictly increasing and convex in t for $t_1^* \le t < t_2^*$, and A_t is strictly increasing and concave in t for $t \ge t_2^*$. If the first breakdown occurs at time t, adoption ceases from then on.*

Good News. *In the unique equilibrium of Theorem 2, $A_t = 1 - e^{-\rho t}$ is strictly increasing and concave for all $t < t^*$. If there is a breakthrough prior to t^*, then $A_t = 1 - e^{-\rho t}$ for all t. If the first breakthrough occurs at $s > t^*$ (which requires $\varepsilon > 0$), then adoption comes to a temporary standstill between t^* and s, and for all $t \ge s$, A_t is strictly increasing and concave, and is given by $1 - e^{-\rho(t^*+t-s)}$.*

Thus, in bad news markets (Figure 2, left), the adoption curve exhibits an S-shaped (i.e., convex–concave) growth pattern whenever $t_1^* < t_2^*$, where convex growth coincides with the partial adoption region (t_1^*, t_2^*). By contrast, in good news markets (Figure 2, right), adoption proceeds in (up to two) concave bursts. Concave adoption curves also arise in bad news markets with very optimistic and impatient consumers or little potential for social learning (so that $t_1^* = t_2^*$ by Lemma 2).

The fact that the convex growth period of A_t under bad news coincides with the partial adoption region (t_1^*, t_2^*) is tied to consumer indifference in this region. Absent breakdowns, consumers grow increasingly optimistic about the quality of the innovation, which increases their opportunity cost of delaying adoption. To maintain indifference, the benefit to delaying adoption must then also increase over time. This is achieved by increasing the arrival rate of future breakdowns, which improves the odds that waiting will allow consumers to avoid the bad product. Since the arrival rate of information is increasing in the flow n_t of new adopters, this means that n_t must strictly increase throughout (t_1^*, t_2^*), i.e., that A_t is convex.[25] By contrast, the concave growth regions under both bad and good news simply reflect the gradual depletion of the population when all consumers adopt immediately upon an opportunity.[26]

[25]This argument for convex growth does not rely on the linearity of λn_t; it remains valid as long as the rate at which the bad product generates breakdowns at t is increasing in n_t.

[26]If there is an inflow of new consumers of $i_t = \gamma N_t$ at all t (i.e., the population size grows exponentially at rate γ absent adoption), then it can be shown that adoption is eventually concave if and only if the growth rate γ is less than the rate ρ of stochastic adoption opportunities.

As discussed in the Introduction, S-shaped adoption is documented for many innovations. Our model complements existing explanations (see Section 1.1) by identifying a purely informational channel for this regularity: If there is a high enough chance that previous adopters' experiences may reveal negative information about the innovation and consumers are forward-looking, then S-shaped adoption can arise due to some consumers strategically delaying adoption. This channel may be especially natural for innovations whose introduction is accompanied by substantial safety concerns, as may plausibly be the case for our motivating example of new medical procedures, where S-shaped adoption patterns are indeed commonly documented.[27]

Though less prevalent than S-shaped curves, concave adoption is another leading pattern documented in the marketing literature (e.g., Keillor (2007), pp. 51–61), with leisure-enhancing innovations such as movies, books, and games as examples. While our model abstracts away from many important product-specific forces, Corollary 1 suggests some factors that could contribute to concave adoption. In particular, high levels of consumer impatience or optimism, or if social learning in these markets is predominantly via good news signals or their absence (as Section 2.2 suggested could be driven by features of the relevant review platforms).

4.2 *The effect of increased opportunities for social learning*

Next, we consider an increase in the potential for social learning $\Lambda_0 := \lambda N_0$, capturing either a greater ease of information transmission (e.g., due to the introduction of new social networking platforms) or a larger community of consumers. We ask how this affects welfare, learning, and adoption dynamics. Again, informational free-riding in the form of partial adoption has important implications. Indeed, under bad news, an economy's ability to harness its potential for social learning is subject to a *saturation effect*: If the equilibrium features partial adoption, then further increases in the potential for social learning are welfare-neutral, cause learning to slow down over certain periods, and decrease adoption levels at all times.

Formally, we fix all other parameters and study the effect of increasing Λ_0 on ex ante equilibrium welfare $W_0(\Lambda_0)$, no-news posteriors $p_t^{\Lambda_0}$, and ex ante expected adoption levels $A_t(\Lambda_0, G)$ and $A_t(\Lambda_0, B)$ conditional on good and bad quality, respectively. We assume that the original potential for social learning Λ_0 is such that there is partial adoption, i.e., $t_1^*(\Lambda_0) < t_2^*(\Lambda_0)$; under the conditions in Lemma 2, this is the case whenever Λ_0 is large enough.

Proposition 1. *Consider learning via bad news. Fix r, ρ, ε, and p_0. If Λ_0 is such that $t_1^*(\Lambda_0) < t_2^*(\Lambda_0)$, then an increase in the potential for social learning to $\hat{\Lambda}_0 > \Lambda_0$ has the following effect:*

[27] See, e.g., the adoption data for bariatric surgery in Buchwald and Oien (2009, 2013).

(i) ***Welfare Neutrality.*** *We have* $W_0(\hat{\Lambda}_0) = W_0(\Lambda_0)$.

(ii) ***Non-Monotonicity of Learning.*** *There exists* $\overline{t} > t_2^*(\Lambda_0)$ *such that*

$$\begin{cases} p_t^{\Lambda_0} = p_t^{\hat{\Lambda}_0} & \text{if } t \leq t_2^*(\Lambda_0) \quad \text{(learning is equally fast under } \Lambda_0 \text{ and } \hat{\Lambda}_0\text{)} \\ p_t^{\Lambda_0} > p_t^{\hat{\Lambda}_0} & \text{if } t_2^*(\Lambda_0) < t < \overline{t} \quad \text{(learning is slower under } \hat{\Lambda}_0\text{)} \\ p_t^{\Lambda_0} < p_t^{\hat{\Lambda}_0} & \text{if } t > \overline{t} \quad \text{(learning is faster under } \hat{\Lambda}_0\text{).} \end{cases}$$

(iii) ***Slowdown of Adoption.*** *For all* t *and* $\theta = B, G$, *we have* $A_t(\Lambda_0, \theta) \geq A_t(\hat{\Lambda}_0, \theta)$, *with strict inequality for all* $t > t_1^*(\Lambda_0)$.

We prove Proposition 1 in Appendix A.6. The idea behind (i) is as follows. Since the equilibrium features partial adoption at Λ_0, the same is true when the potential for social learning increases to $\hat{\Lambda}_0$. Moreover, both the time t_1^* at which adoption begins and the posterior $p_{t_1^*}$ at t_1^* are the same under Λ_0 and $\hat{\Lambda}_0$.[28] Since consumers strictly prefer to delay at all $t < t_1^*$, and are indifferent between delaying and adopting at t_1^*, ex ante welfare under both Λ_0 and $\hat{\Lambda}_0$ then corresponds to the expected payoff to waiting until t_1^* and adopting at t_1^* absent breakdowns. Thus, $W_0(\hat{\Lambda}_0) = W_0(\Lambda_0)$.[29]

This welfare neutrality result contrasts with the cooperative benchmark where consumers coordinate on socially optimal adoption levels. In the latter case, increased opportunities for social learning are strictly beneficial and for any $p_0 > \frac{1}{2}$, the first-best (complete information) payoff of $\frac{\rho}{r+\rho} p_0$ can be approximated in the limit as $\Lambda_0 \to \infty$.[30] The result also contrasts with myopic social learning or forward-looking exogenous learning, where welfare necessarily increases in response to more informative signals (even if consumers are heterogeneous).[31]

Points (ii) and (iii) further illuminate the forces behind welfare neutrality. By (ii), an increase in Λ_0 affects learning dynamics in a non-monotonic manner. Thus, the impact on a consumer's expected payoff varies with the time t at which he obtains his first adoption opportunity. If $t \leq t_2^*(\Lambda_0)$, his expected payoff is the same under Λ_0 and $\hat{\Lambda}_0$. If $t \in (t_2^*(\Lambda_0), \overline{t})$, he is worse off under $\hat{\Lambda}_0$, because in case the innovation is bad, he is less likely to have found out by then than under Λ_0.[32] Finally, if $t > \overline{t}$, he is better off under $\hat{\Lambda}_0$. Depending on $\hat{\Lambda}_0$, $\overline{t}$ adjusts endogenously to balance out the benefits, which arrive at times after $\overline{t}$, with the costs incurred at times $(t_2^*(\Lambda_0), \overline{t})$.

[28]Indeed, as we saw in Section 3.2, t_1^* is the first time at which the posterior exceeds the threshold $\overline{p} = \frac{\varepsilon+r}{\varepsilon+2r}$ and learning up to t_1^* is purely via the exogenous news source.

[29]Related welfare neutrality results can arise in mixed equilibria in other games; e.g., in certain static public goods provision games, the equilibrium welfare/provision probability of the public good can be independent of the number of players.

[30]Frick and Ishii (2023) (Supplement C) show the cooperative benchmark is all-or-nothing, with no (resp. immediate) adoption below (resp. above) a cutoff belief p^{SO}. Equilibrium adoption displays two inefficiencies: (i) it starts too late ($p^{SO} < p_{t_1^*}$); (ii) once it starts it is initially too low (if $t_1^* < t_2^*$).

[31]To define ex ante welfare with myopic consumers, assume that consumers' payoffs are discounted at some arbitrary rate $r > 0$, but consumers behave myopically.

[32]The fact that learning on $(t_2^*(\Lambda_0), \overline{t})$ is slower under $\hat{\Lambda}_0$ than Λ_0 reflects that the flow of adopters under Λ_0 jumps up at $t_2^*(\Lambda_0)$ (due to the transition from the partial adoption to immediate adoption regions), whereas under $\hat{\Lambda}_0$, partial adoption continues until $t_2^*(\hat{\Lambda}_0) > t_2^*(\Lambda_0)$.

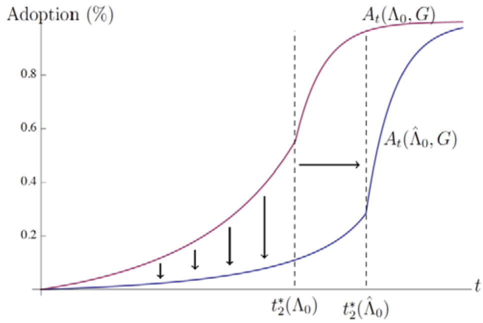

FIGURE 3. The effect of increased opportunities for social learning on the adoption of a good product under bad news ($\hat{\Lambda}_0 > \Lambda_0$).

Similarly, by (iii), an increase in Λ_0 strictly decreases the adoption $A_t(\Lambda_0, G)$ of good products (which is harmful), but also decreases the ex ante expected adoption $A_t(\Lambda_0, B)$ of bad products (which is beneficial), and welfare neutrality obtains because these forces balance out in equilibrium. Figure 3 illustrates that the strict slowdown in the adoption of good products is due to two effects: On the extensive margin, the increase in Λ_0 pushes out t_2^*, i.e., prolongs free-riding; on the intensive margin, the increase drives down the growth rate of A_t at all $t < t_2^*(\Lambda_0)$.

Point (iii) yields new testable implications relative to existing models of innovation adoption, suggesting, for example, that the fraction of adopters may grow more slowly in larger communities. Broadly consistent with this, Bandiera and Rasul (2006) study the adoption of a new crop by farmers in Mozambique and find that farmers whose network includes many adopters may be less likely to adopt initially themselves; thus, in equilibrium, larger networks of farmers should feature lower percentages of adoption.[33]

Finally, the logic behind the saturation effect relies crucially on partial adoption/informational free-riding. If under bad news, Λ_0 is so low that there is no partial adoption in equilibrium, then increasing Λ_0 is strictly beneficial (see Frick and Ishii (2023), Supplement B.1). Likewise, there is no saturation effect under good news (see Frick and Ishii (2023), Supplement B.2): Since equilibrium adoption is all-or-nothing, increasing the potential for social learning speeds up learning at all times, which strictly improves welfare (provided $\varepsilon > 0$).[34]

REMARK 1. Proposition 1 shows that increasing Λ_0 is welfare-neutral under bad news. More strongly, if consumers have heterogeneous discount rates, then increasing the potential for social learning can lead to Pareto decreases in ex ante welfare. To illustrate,

[33]In related work, Munshi (2004) finds that in rice-growing regions in India, where (due to more heterogeneous plot conditions) social learning is less feasible than in wheat-growing areas, farmers are more likely to experiment with a new crop than their counterparts in wheat-growing areas.

[34]Even under good news, increasing Λ_0 increases welfare only if this affects agents' preference for adoption vs. delay at some histories. If $\varepsilon = 0$, agents weakly prefer to adopt at all histories (note $u_t = W_t$ for all $t \geq t^*$ as $p_t = p^s = \frac{1}{2}$ for all $t \geq t^*$); hence, $W_0(\Lambda_0) = \frac{\rho}{r+\rho}(2p_0 - 1)$ is independent of Λ_0. If $\varepsilon > 0$, increasing Λ_0 improves welfare by leading more agents to adopt only after a breakthrough ($u_t < W_t$ for all $t > t^*$ and t^* is decreasing in Λ_0).

suppose $\varepsilon = 0$ and introduce a single (mass 0) impatient agent with discount rate $r_i > r$ into the population.[35] Then, under the assumptions in Proposition 1, increasing λ to $\hat{\lambda}$ is welfare-neutral for the original population, but makes this impatient agent strictly worse off. Indeed, since the patient agents are initially indifferent between adopting and delaying, the impatient agent adopts upon his first opportunity absent breakdowns in both environments. By the non-monotonicity of learning in Proposition 1, there exists some time $\bar{t} > t^* := t_2^*(\lambda)$ such that learning is strictly slower under $\hat{\lambda}$ between t^* and $\bar{t}$, but faster from time $\bar{t}$ on (and learning is equally fast under λ, $\hat{\lambda}$ up to t^*). For patient agents, the costs of the early deceleration in learning and the benefits of the later acceleration exactly balance out. However, the impatient agent is hurt, because relative to a patient agent, he weights the early costs more heavily than the later benefits. ▲

5. Concluding remarks

This paper develops a model of innovation adoption when consumers are forward-looking and learning is social. Our analysis isolates the effect of purely informational incentives on aggregate adoption dynamics, learning, and welfare. We highlight how qualitative and quantitative features of the learning environment shape these incentives, most importantly by determining whether or not there is informational free-riding in the form of partial adoption. The presence or absence of partial adoption has observable implications, suggesting a novel channel for two widespread adoption patterns: S-shaped and concave curves. Moreover, partial adoption has important welfare implications, entailing that increased opportunities for social learning need not benefit consumers and can be strictly harmful. Below, we briefly comment on some modifications and extensions of our model.

Adoption opportunities. We assumed that consumers receive adoption opportunities at an arbitrarily large but finite Poisson rate ρ. This avoided technical issues related to defining strategies and continuation payoffs when agents can move continuously and adoption processes can feature mass points. The key qualitative implication of a finite ρ in both the bad and good news equilibrium is to generate concave adoption regions. To illustrate what happens as $\rho \to \infty$, suppose $\varepsilon = 0$ and $p_0 > \frac{1}{2}$. Under bad news, the immediate (i.e., concave) adoption phase disappears as $\rho \to \infty$. In Figure 1, $\lim_{\rho\to\infty} N^*(p) = 0$ for all $p < 1$, so region I vanishes.[36] Thus, by Theorem 1, there is an initial partial adoption phase with flow of adopters $n_t = \frac{r(2p_t-1)}{\lambda(1-p_t)}$ and, in the limit as $\rho \to \infty$, this phase continues all the way until the finite time t_2^* at which the population is fully depleted.[37] Under good news, Theorem 2 implies that for any finite ρ, equilibrium is all-or-nothing with cutoff posterior $p^s = \frac{1}{2}$, but as $\rho \to \infty$, the time t^* it takes to

[35] Section 4.3 of Frick and Ishii (2015) instead considered a small mass of impatient consumers.

[36] Intuitively, if there is any positive mass M_t of immediate adopters, then it is strictly beneficial to wait an instant, as the cost of delaying the decision by an instant is negligible (of order dt) relative to the probability $(1 - p_t)(1 - e^{-\lambda M_t})$ of observing a breakdown and avoiding the bad product.

[37] To see why t_2^* is finite, note that the ODE for partial adoption implies $n_t = \frac{r}{\lambda}\frac{2p_0-1}{e^{-rt}p_0-(2p_0-1)}$, which tends to ∞ by the finite time $t = \frac{1}{r}\ln\frac{p_0}{2p_0-1}$. We also note that parts (i) and (iii) of Proposition 1 remain valid as $\rho \to \infty$, but the non-monotonicity of learning in part (ii) no longer arises in the limit, because the acceleration/deceleration in learning occurs during the immediate adoption phase.

reach p^s absent news tends to 0. Thus, the initial concave adoption region approximates a single mass point of $M_0 = \frac{1}{\lambda} \ln \frac{p_0}{1-p_0}$ adopters, where M_0 is such that absent news, the belief jumps down to p^s. Hence, as $\rho \to \infty$, the good news equilibrium approximates an initial burst of partial adoption (followed by a second burst if there is a breakthrough), but a drawn-out region of partial adoption can still only arise under bad news.[38]

Learning from the stock of adopters. In our model, the social learning component of the signal arrival rate at time t, λn_t, depends only on the flow n_t of new adopters. This effectively assumes that adopters can generate signals only once, at the time of adoption, approximating settings where the probability of receiving signals about the quality of the innovation (e.g., complications from a new medical procedure) depreciates rapidly from the time of adoption. In contrast, for some durable goods, it may be more natural to let signals at t arrive at rate λS_t, where $S_t := \int_0^t n_s \, ds$ represents the stock of adopters, capturing that adopters can generate signals repeatedly over time. This would produce similar results. Specifically, similar arguments yield the existence and uniqueness of equilibrium under both bad and good news. The good news equilibrium is again all-or-nothing, while, for appropriate parameters, the bad news equilibrium again features a partial adoption region with behavior pinned down by the indifference condition $S_t = \frac{r(2p_t-1)}{\lambda(1-p_t)} - \frac{\varepsilon}{\lambda}$. Finally, the partial adoption region again exhibits convex growth in adoption levels.[39]

More general signal processes. As in many applications of Poisson learning, we have focused for tractability on conclusive bad or good news signals. While a careful investigation of more general signal processes is beyond the scope of this paper, the analysis extends readily to hybrid environments with two types of conclusive Poisson signals: bad news *and* good news signals with respective arrival rates $\lambda_B n_t$ and $\lambda_G n_t$. In particular, if $\lambda_B > \lambda_G$, the equilibrium is analogous to Theorem 1. Some of our insights also extend beyond environments with conclusive signals. For example, we note that partial adoption relies crucially on the possibility of news events that trigger discrete downward jumps in beliefs (although such events need not conclusively signal bad quality). Without such events (e.g., when learning is based on inconclusive good news Poisson

[38]If, instead, each consumer's first adoption opportunity arrives at rate $\rho < \infty$, but subsequent adoption opportunities arrive continuously, the good news equilibrium is still all-or-nothing as in Theorem 2, except that the cutoff belief $\lim_{\rho\to\infty} p^s = \frac{\varepsilon+r}{\varepsilon+2r}$ is greater than p^s (if $\varepsilon > 0$). The bad news equilibrium is qualitatively unchanged: Under suitable parameters, there is an initial partial adoption region with convex adoption growth (which continues until the stock of consumers who have received a first adoption opportunity is depleted); from then on, the remaining consumers adopt immediately at their first opportunity (leading to concave growth). However, the non-monotonicity of learning in Proposition 1(ii) no longer arises, as the flow of adopters now features a downward jump at the transition from partial to immediate adoption.

[39]Indeed, as in Section 3.2, indifference requires the benefit of avoiding a bad product when a breakdown occurs ($(1-p_t)(\lambda S_t + \varepsilon)$) to equal the cost of delaying adoption absent news ($r(2p_t - 1)$). Since consumers grow more optimistic absent news, this has two implications throughout the indifference region: (i) beliefs p_t increase convexly, as the growth rate of p_t equals the instantaneous probability of a breakdown ($\frac{\dot{p}_t}{p_t} = (1-p_t)(\lambda S_t + \varepsilon)$), which must increase over time to balance out the increasing cost of delay; (ii) the stock of adopters $S_t = S(p_t)$ increases convexly as a function of p_t, to ensure that breakdowns arrive at a rate that counterbalances the convex growth (with respect to p_t) of the ratio $\frac{r(2p_t-1)}{(1-p_t)}$ between the cost of delay and the probability of facing a bad product. Combining (i) and (ii), it follows that S_t—and, hence, adoption levels—increases convexly over time.

or Brownian signals), a similar logic as in Section 3.3 implies that there cannot be continuous regions of partial adoption, because a consumer who is willing to adopt cannot instantaneously acquire decision-relevant information (see Frick and Ishii (2023), Supplement D).[40]

Appendix: Proofs

A.1 *Preliminary lemmas*

The following five lemmas will be used throughout the Appendix. For any feasible adoption flow (n_t), we denote by (W_t) the corresponding no-news value to waiting and denote by (p_t) the no-news posterior, without making explicit the dependency on (n_t).

Lemma A.1. *For any feasible adoption flow (n_t), the corresponding (W_t) and (p_t) are continuous in t.*

The proof is immediate from the definitions of p_t and W_t in Sections 2.2 and 2.3.

Lemma A.2. *Suppose that (n_s) is an equilibrium and that $W_t < 2p_t - 1$ for some $t > 0$. Then there exists $\nu > 0$ such that (W_τ) is continuously differentiable in τ on the interval $(t-\nu, t+\nu)$ and for all $\tau \in (t-\nu, t+\nu)$,*

$$\dot{W}_\tau = \big(r + \rho + (\varepsilon_G + \lambda_G \rho N_\tau) p_\tau + (\varepsilon_B + \lambda_B \rho N_\tau)(1 - p_\tau)\big) W_\tau$$
$$- \rho(2p_\tau - 1) - p_\tau(\varepsilon_G + \lambda_G \rho N_\tau) \frac{\rho}{\rho + r}.$$

Proof. Suppose $W_t < 2p_t - 1$ for some $t > 0$. Since (W_τ) and (p_τ) are continuous in τ (Lemma A.1), there exists $\nu > 0$ such that $W_\tau < 2p_\tau - 1$ for all $\tau \in (t-\nu, t+\nu)$. Because (n_s) is an equilibrium, this implies that $n_\tau = \rho N_\tau$ for all $\tau \in (t-\nu, t+\nu)$. Thus, n_τ is continuous at all $\tau \in (t-\nu, t+\nu)$. Then W_τ is continuously differentiable in τ for all $\tau \in (t-\nu, t+\nu)$, as

$$W_\tau = \int_\tau^{t+\nu} \rho e^{-(\rho+r)(s-\tau)} \big(p_\tau e^{-\int_\tau^s (\varepsilon_G + \lambda_G n_x)\, dx} - (1-p_\tau) e^{-\int_\tau^s (\varepsilon_B + \lambda_B n_x)\, dx}\big)\, ds$$
$$+ e^{-(r+\rho)(t+\nu-\tau)} \big(p_\tau e^{-\int_\tau^{t+\nu} (\varepsilon_G + \lambda_G n_x)\, dx} + (1-p_\tau) e^{-\int_\tau^{t+\nu} (\varepsilon_B + \lambda_B n_x)\, dx}\big) W_{t+\nu}$$
$$+ \int_\tau^{t+\nu} \rho e^{-(\rho+r)(s-\tau)} p_\tau \big(1 - e^{-\int_\tau^s (\varepsilon_G + \lambda_G n_x)\, dx}\big)\, ds$$
$$+ e^{-(r+\rho)(t+\nu-\tau)} p_\tau \big(1 - e^{-\int_\tau^{t+\nu} (\varepsilon_G + \lambda_G n_x)\, dx}\big) \frac{\rho}{\rho + r}.$$

[40] In contrast, Laiho, Murto, and Salmi (2024) obtain partial adoption/gradualism in a model with Brownian learning from the stock of adopters and continuous adoption opportunities.

The derivative of W_τ can be computed using Ito's lemma for processes with jumps. Given perfect Poisson learning, the derivation is simple and we provide it for completeness. As above, for any $\Delta \in (0, t+\nu-\tau)$, we can rewrite W_τ as

$$W_\tau = \int_\tau^{\tau+\Delta} \rho e^{-(\rho+r)(s-\tau)}\big(p_\tau e^{-\int_\tau^s(\varepsilon_G+\lambda_G n_x)\,dx} - (1-p_\tau)e^{-\int_\tau^s(\varepsilon_B+\lambda_B n_x)\,dx}\big)\,ds$$
$$+ e^{-(r+\rho)\Delta}\big(p_\tau e^{-\int_\tau^{\tau+\Delta}(\varepsilon_G+\lambda_G n_x)\,dx} + (1-p_\tau)e^{-\int_\tau^{\tau+\Delta}(\varepsilon_B+\lambda_B n_x)\,dx}\big)W_{\tau+\Delta}$$
$$+ \int_\tau^{\tau+\Delta} \rho e^{-(r+\rho)(s-\tau)} p_\tau\big(1 - e^{-\int_\tau^s(\varepsilon_G+\lambda_G n_x)\,dx}\big)\,ds$$
$$+ e^{-(r+\rho)\Delta} p_\tau\big(1 - e^{-\int_\tau^{\tau+\Delta}(\varepsilon_G+\lambda_G n_x)\,dx}\big)\frac{\rho}{\rho+r}.$$

Since this is true for all $\Delta \in (0, t+\nu-\tau)$, the right-hand side of this identity, which we denote R_Δ, is continuously differentiable with respect to Δ and satisfies $\frac{d}{d\Delta}R_\Delta \equiv 0$. Taking the limit as $\Delta \to 0$ and since $\dot{W}_\tau = \lim_{\Delta\to 0}\frac{d}{d\tau}W_{\tau+\Delta}$ by continuous differentiability, we then obtain

$$\dot{W}_\tau = \big(r+\rho+(\varepsilon_G+\lambda_G n_\tau)p_\tau + (\varepsilon_B+\lambda_B n_\tau)(1-p_\tau)\big)W_\tau$$
$$- \rho(2p_\tau - 1) - p_\tau(\varepsilon_G+\lambda_G n_\tau)\frac{\rho}{\rho+r}.$$

Plugging in $n_\tau = \rho N_\tau$ yields the desired expression. □

Lemma A.3. *Suppose that* (n_τ) *is an equilibrium and that* $W_t > 2p_t - 1$ *for some* $t > 0$. *Then there exists* $\nu > 0$ *such that* (W_τ) *is continuously differentiable in* τ *on the interval* $(t-\nu, t+\nu)$ *and for all* $\tau \in (t-\nu, t+\nu)$,

$$\dot{W}_\tau = \big(r + p_\tau\varepsilon_G + (1-p_\tau)\varepsilon_B\big)W_\tau - p_\tau\varepsilon_G\frac{\rho}{\rho+r}.$$

Proof. The proof follows the same lines as that of Lemma A.2. Lemma A.1 again implies that if $W_t > 2p_t - 1$, then there exists $\nu > 0$ such that $W_\tau > 2p_\tau - 1$ for all $\tau \in (t-\nu, t+\nu)$. By the definition of equilibrium, $n_\tau = 0$ for all $\tau \in (t-\nu, t+\nu)$.

Hence, W_τ satisfies

$$W_\tau = e^{-r(t+\nu-\tau)}\big(p_\tau e^{-\varepsilon_G(t+\nu-\tau)} + (1-p_\tau)e^{-\varepsilon_B(t+\nu-\tau)}\big)W_{t+\nu}$$
$$+ p_\tau \int_\tau^{t+\nu} \varepsilon_G e^{-(\varepsilon_G+r)s}\frac{\rho}{\rho+r}\,ds$$

and, thus, is continuously differentiable in τ.

To compute the derivative, note again that for any $\Delta \in (0, t+\nu-\tau)$,

$$W_\tau = e^{-r\Delta}\big(p_\tau e^{-\varepsilon_G \Delta} + (1-p_\tau)e^{-\varepsilon_B \Delta}\big)W_{t+\Delta} + p_\tau \int_\tau^{\tau+\Delta} \varepsilon_G e^{-(\varepsilon_G+r)s}\frac{\rho}{\rho+r}\,ds.$$

Differentiating both sides with respect to Δ and taking the limit as $\Delta \to 0$,

$$\dot{W}_\tau = \big(r + p_\tau\varepsilon_G + (1-p_\tau)\varepsilon_B\big)W_\tau - p_\tau\varepsilon_G\frac{\rho}{\rho+r},$$

as claimed. □

Lemma A.4. *Suppose* (n_t) *is an equilibrium under bad news. Suppose* $\varepsilon > 0$ *or* $p_0 > \frac{1}{2}$. *Then* $\lim_{t\to\infty} p_t = \mu(\varepsilon, \Lambda_0, p_0)$ *and* $\lim_{t\to\infty} W_t = \frac{\rho}{\rho+r}(2\mu(\varepsilon, \Lambda_0, p_0) - 1)$, *where*

$$\mu(\varepsilon, \Lambda_0, p_0) := \begin{cases} 1 & \text{if } \varepsilon > 0 \\ \dfrac{p_0}{p_0 + (1-p_0)e^{-\Lambda_0}} & \text{if } \varepsilon = 0. \end{cases}$$

Proof. Suppose first that $\varepsilon > 0$. Then trivially $p_t \to 1$ as $t \to \infty$. Since for any t, $\frac{\rho}{\rho+r}(2p_t - 1) \le W_t \le \frac{\rho}{\rho+r}$, this implies that $\lim_{t\to\infty} W_t = \frac{\rho}{\rho+r}$, as claimed.

Now suppose $\varepsilon = 0$ and $p_0 > 1/2$. Note that $W_t \le 2p_t - 1$ for all t. Indeed, suppose $W_t > 2p_t - 1$ for some t. If $W_s > 2p_s - 1$ for all $s \ge t$, then $W_t = 0$, contradicting $W_t > 2p_t - 1 \ge 2p_0 - 1 > 0$. Thus, we can find $s > t$ such that $W_s = 2p_s - 1$ and $W_{s'} > 2p_{s'} - 1$ for all $s' \in (t, s)$. This implies $n_{s'} = 0$ for all s', and, hence, $W_t = e^{-r(s-t)}W_s = e^{-r(s-t)}(2p_s - 1) = e^{-r(s-t)}(2p_t - 1)$, again contradicting $W_t > 2p_t - 1 > 0$.

Let $N^* := \lim_{t\to\infty}\int_0^t n_s\,ds = \sup_t \int_0^t n_s\,ds \le N_0$. Let $p^* := \lim_{t\to\infty} p_t = \sup_t p_t$. For any $\nu > 0$, we can find t^* such that whenever $t > t^*$, then $e^{-\lambda\int_{t^*}^t n_s\,ds} > 1-\nu$. Because $2p_t - 1 \ge W_t$ for all t, we can then write the value to waiting at all $t > t^*$ as

$$\begin{aligned} W_t &= \int_t^\infty \rho e^{-(r+\rho)\tau}\big(p_t - (1-p_t)e^{-\lambda\int_t^\tau n_s\,ds}\big)\,d\tau \\ &\le \frac{\rho}{r+\rho}\big(p_t - (1-p_t)(1-\nu)\big). \end{aligned}$$

By optimality, $W_t \ge \frac{\rho}{\rho+r}(2p_t - 1)$ for all t, so by combining, we have

$$\frac{\rho}{\rho+r}\big(2p^* - 1\big) \le \lim_{t\to\infty}\inf W_t \le \lim_{t\to\infty}\sup W_t \le \frac{\rho}{r+\rho}\big(p^* - \big(1-p^*\big)(1-\nu)\big).$$

Since this is true for all $\nu > 0$, it follows that

$$\lim_{t\to\infty} W_t = \frac{\rho}{r+\rho}\big(2p^* - 1\big),$$

which is strictly less than $2p^* - 1$, so for all t sufficiently large we must have $2p_t - 1 > W_t$. Then for all t sufficiently large, we have $n_t = \rho N_t$. Thus, $N^* = N_0$ and, therefore, $p^* = \mu(\varepsilon, \Lambda_0, p_0)$. □

LEMMA A.5. *Suppose that learning is via bad news. Suppose that $\varepsilon = 0$ and $p_0 \leq \frac{1}{2}$. Then the unique equilibrium satisfies $n_t = 0$ for all t.*

PROOF. Suppose that (n_s) is an equilibrium and suppose, for a contradiction, that $t_1^* := \inf\{t : n_t > 0\} < \infty$. Pick $t \geq t_1^*$ such that $n_t > 0$. By right-continuity of (n_s), we have $n_\tau > 0$ for all $\tau > t$ sufficiently close to t. This implies

$$\int_{t_1^*}^{\infty} \rho e^{-(r+\rho)(s-t)} \left(p_{t_1^*} - (1 - p_{t_1^*}) e^{-\int_{t_1^*}^{s} \lambda n_k \, dk}\right) ds > \frac{\rho}{r+\rho}(2p_{t_1^*} - 1) \geq 2p_{t_1^*} - 1, \tag{6}$$

where the second inequality holds because $p_{t_1^*} = p_0 \leq \frac{1}{2}$. The integral on the left-hand side is the expected payoff at time t_1^* to adopting at the first opportunity in the future, conditional on no breakdown having occurred prior to this opportunity. By optimality of the value to waiting, this is weakly less than $W_{t_1^*}$. Hence, (6) implies $W_{t_1^*} > 2p_{t_1^*} - 1$. By continuity of (W_s) and (p_s), it follows that for all $s \geq t_1^*$ sufficiently close to t_1^*, $W_s > 2p_s - 1$ and, hence, $n_s = 0$, contradicting the definition of t_1^*.

This leaves $n_t = 0$ for all t as the only candidate equilibrium. In this case, $W_t = 0 \geq 2p_0 - 1 = 2p_t - 1$ for all t, so this is indeed an equilibrium. □

A.2 *Proof of Lemma 1*

Good News. Suppose first that learning is via good news.

Step 1: $W_t = 2p_t - 1 \Longrightarrow W_\tau \geq 2p_\tau - 1$ for all $\tau \geq t$. Suppose $W_t = 2p_t - 1$ at some time t and suppose, for a contradiction, that at some time $s' > t$, we have $W_{s'} < 2p_{s'} - 1$. Let $s^* := \sup\{s < s' : W_s = 2p_s - 1\}$.

By continuity, $s^* < s'$, $W_{s^*} = 2p_{s^*} - 1$, and $W_s < 2p_s - 1$ for all $s \in (s^*, s')$. Then by Lemma A.2, the right-hand derivative of $W_s - (2p_s - 1)$ at s^* exists and satisfies

$$\lim_{s \downarrow s^*} \dot{W}_s - 2\dot{p}_s = r(2p_{s^*} - 1) + p_{s^*}(\varepsilon + \lambda\rho N_{s^*})\frac{r}{\rho + r} > 0.$$

This implies that for some $s \in (s^*, s')$ sufficiently close to s^*, we have $W_s > 2p_s - 1$, which is a contradiction.

Step 2: $W_t > 2p_t - 1 \Longrightarrow W_\tau > 2p_\tau - 1$ for all $\tau > t$. Suppose, for a contradiction, that there exists $s' > t$ such that $W_{s'} = 2p_{s'} - 1$. Let $s^* := \inf\{s > t : W_s = 2p_s - 1\}$. By continuity, $s^* > t$, $W_{s^*} = 2p_{s^*} - 1$, and $W_s > 2p_s - 1$ for all $s \in (t, s^*)$. Note that $p_{s^*} \geq \frac{1}{2}$, because W_{s^*} is bounded below by 0. Moreover, by Lemma A.3, the left-hand derivative of $W_s - (2p_s - 1)$ at s^* exists and is given by

$$\lim_{s \uparrow s^*} \dot{W}_s - 2\dot{p}_s = r(2p_{s^*} - 1) + p_{s^*}\frac{r}{\rho + r}\varepsilon.$$

If $\varepsilon > 0$, this is strictly positive, implying that for some $s \in (t, s^*)$ sufficiently close to s^*, we have $W_s < 2p_s - 1$, which is a contradiction. If $\varepsilon = 0$, then for all $s \in (t, s^*)$, we have $p_{s^*} = p_s$ and $W_s = e^{-r(s^*-s)}W_{s^*} = e^{-r(s^*-s)}(2p_{s^*} - 1) \leq 2p_{s^*} - 1$. Thus, $W_s \leq 2p_s - 1$, again contradicting $W_s > 2p_s - 1$.

Bad News. Now, suppose learning is via bad news. If $\varepsilon = 0$ and $p_0 \le \frac{1}{2}$, then, by Lemma A.5, $n_t = 0$ for all t, so the proof is trivial. Thus, suppose that either $\varepsilon > 0$ or $p_0 > \frac{1}{2}$.

Step 1: $W_t = 2p_t - 1 \Longrightarrow W_\tau \le 2p_\tau - 1$ for all $\tau \ge t$. Suppose that $W_t = 2p_t - 1$ and suppose, for a contradiction, that $W_{s'} > 2p_{s'} - 1$ for some $s' > t$. Let $\overline{s} := \inf\{s > s' : W_t \le 2p_s - 1\} < \infty$, since, by Lemma A.4, $\lim_{t\to\infty} 2p_t - 1 > \lim_{t\to\infty} W_t$. Let $\underline{s} := \sup\{s < s' : W_s \le 2p_s - 1\}$. Then $\underline{s} < \overline{s}$, $W_{\underline{s}} = 2p_{\underline{s}} - 1$, $W_{\overline{s}} = 2p_{\overline{s}} - 1$, and $W_s > 2p_s - 1$ for all $s \in (\underline{s}, \overline{s})$. Lemma A.3 together with the fact that $n_s = 0$ for all $s \in (\underline{s}, \overline{s})$ implies the two limits

$$L_{\underline{s}} := \lim_{s\downarrow\underline{s}}\left(\dot{W}_s - \frac{d}{ds}(2p_s - 1)\right) = \big(r + (1 - p_{\underline{s}})\varepsilon\big)(2p_{\underline{s}} - 1) - 2p_{\underline{s}}(1 - p_{\underline{s}})\varepsilon$$

$$L_{\overline{s}} := \lim_{s\uparrow\overline{s}}\left(\dot{W}_s - \frac{d}{ds}(2p_s - 1)\right) = \big(r + (1 - p_{\overline{s}})\varepsilon\big)(2p_{\overline{s}} - 1) - 2p_{\overline{s}}(1 - p_{\overline{s}})\varepsilon.$$

Because $W_s > 2p_s - 1$ for all $s \in (\underline{s}, \overline{s})$, we need $L_{\underline{s}} \ge 0$ and $L_{\overline{s}} \le 0$. Rearranging yields

$$r(2p_{\underline{s}} - 1) \ge (1 - p_{\underline{s}})\varepsilon \quad \text{and} \quad r(2p_{\overline{s}} - 1) \le (1 - p_{\overline{s}})\varepsilon.$$

If $\varepsilon > 0$, then $p_{\overline{s}} > p_{\underline{s}}$, so this is impossible. On the other hand, if $\varepsilon = 0$ and $p_0 > \frac{1}{2}$, then for all $s \in (\underline{s}, \overline{s})$, we have that $p_s = p_{\overline{s}} > \frac{1}{2}$ and $W_s = e^{-r(\overline{s}-s)}W_{\overline{s}}$. Since $W_{\overline{s}} = 2p_{\overline{s}} - 1$, this implies $W_s = e^{-r(\overline{s}-s)}(2p_s - 1) < 2p_s - 1$, contradicting $W_s > 2p_s - 1$. This completes the proof of Step 1.

Step 2: $W_t < 2p_t - 1 \Longrightarrow W_\tau < 2p_\tau - 1$ for all $\tau > t$. Suppose $W_t < 2p_t - 1$, let $\underline{s} := \inf\{s' > t : W_{s'} \ge 2p_{s'} - 1\}$, and suppose, for a contradiction, that $\underline{s} < \infty$. By continuity, $W_\tau < 2p_\tau - 1$ for all $\tau \in [t, \underline{s})$ and $W_{\underline{s}} = 2p_{\underline{s}} - 1$. Furthermore, by Lemma A.4, there exists some $\overline{s} \ge \underline{s}$ such that $2p_{\overline{s}} - 1 = W_{\overline{s}}$ and $2p_s - 1 > W_s$ for all $s > \overline{s}$. Lemma A.2 implies the two limits

$$H_{\underline{s}} := \lim_{s\uparrow\underline{s}}\left(\dot{W}_s - \frac{d}{ds}(2p_s - 1)\right) = r(2p_{\underline{s}} - 1) - (\varepsilon + \lambda\rho N_{\underline{s}})(1 - p_{\underline{s}})$$

$$H_{\overline{s}} := \lim_{s\downarrow\overline{s}}\left(\dot{W}_s - \frac{d}{ds}(2p_s - 1)\right) = r(2p_{\overline{s}} - 1) - (\varepsilon + \lambda\rho N_{\overline{s}})(1 - p_{\overline{s}}).$$

As usual, because $W_s < 2p_s - 1$ for all $s \in (t, \underline{s})$ and for all $s > \overline{s}$, we must have $H_{\underline{s}} \ge 0$ and $H_{\overline{s}} \le 0$, but since $p_{\overline{s}} \ge p_{\underline{s}}$, this is only possible if $\underline{s} = \overline{s} =: s^*$ and $H_{s^*} = H_{\underline{s}} = H_{\overline{s}} = 0$. Thus,

$$r(2p_{s^*} - 1) = (\varepsilon + \lambda\rho N_{s^*})(1 - p_{s^*}).$$

Now consider any $s \in [t, s^*)$. Because $p_s \le p_{s^*}$ and $N_s \ge N_{s^*}$, we must have

$$r(2p_s - 1) \le (\varepsilon + \lambda\rho N_s)(1 - p_s).$$

Combining this with the fact that $W_s < 2p_s - 1$ yields

$$rW_s < (\varepsilon + \lambda\rho N_s)(1 - p_s) < (2p_s - W_s)(\varepsilon + \lambda\rho N_s)(1 - p_s) + \rho(2p_s - 1 - W_s).$$

Rearranging, we obtain

$$0 < -rW_s + \rho(2p_s - 1 - W_s) + (2p_s - W_s)(\varepsilon + \lambda\rho N_s)(1 - p_s).$$

By Lemma A.2, the right-hand side is the derivative $\frac{d}{ds}(2p_s - 1) - \dot{W}_s$. Thus, for all $s \in [t, s^*)$, $2p_s - 1 > W_s$ and $2p_s - 1 - W_s$ is strictly increasing, contradicting continuity and the fact that $2p_{s^*} - 1 = W_{s^*}$. □

A.3 *Proof of Theorem 1*

Suppose learning is via bad news. Recall the following beliefs defined in Section 3.2:

$$p^s := \frac{(\varepsilon + r)(r + \rho)}{2(\varepsilon + r)(r + \rho) - \varepsilon\rho}, \qquad \overline{p} := \frac{\varepsilon + r}{\varepsilon + 2r}, \qquad p^\sharp := \frac{\rho + r}{\rho + 2r}.$$

Let $p^* := \min\{\overline{p}, p^\sharp\}$. Define the function $G : [0, 1] \times \mathbb{R}_+ \to \mathbb{R}$ by

$$G(p, \Lambda) := \int_0^\infty \rho e^{-(r+\rho)\tau}\big(p - (1 - p)e^{-(\varepsilon\tau + \Lambda(1-e^{-\rho\tau}))}\big)\, d\tau, \quad \text{for all } (p, \Lambda) \in [0, 1] \times \mathbb{R}_+.$$

We extend G to the domain $[0, 1] \times (\mathbb{R}_+ \cup \{\infty\})$ by setting $G(p, \infty) := \frac{\rho}{\rho+r} p$.

Finally, define the nondecreasing function $\Lambda^* : [0, 1] \to \mathbb{R}_+ \cup \{\infty\}$ by

$$\begin{cases} \Lambda^*(p) = 0 & \text{if } p \leq p^s \\ 2p - 1 = G\big(p, \Lambda^*(p)\big) & \text{if } p \in \big(p^s, p^\sharp\big) \\ \Lambda^*(p) = \infty & p \geq p^\sharp. \end{cases}$$

As discussed in the text, for $p \in (p^s, p^\sharp)$, $N^*(p) := \frac{1}{\lambda}\Lambda^*(p)$ has the following property: If $N^*(p)$ consumers remain and if all these remaining consumers adopt at their first future opportunity, then a consumer with current posterior p is indifferent between immediate adoption and adoption at his next opportunity (absent breakdowns). Note that $2p^s - 1 = G(p^s, 0)$ and $G(p^s, 0)$ is the continuation value to adopting at the next opportunity (absent breakdowns) when information arrives purely exogenously, so p^s is the cutoff posterior above which adoption occurs in the single-agent benchmark.

The proof of Theorem 1 proceeds in three steps. Suppose that (n_t) is an equilibrium with associated cutoff times $0 \leq t_1^* \leq t_2^* \leq \infty$ defined by

$$t_1^* := \inf\{t \geq 0 : n_t > 0\}, \qquad t_2^* := \sup\{t \geq 0 : n_t < \rho N_t\}. \tag{7}$$

First, Lemma A.6 shows that if $t_1^* < t_2^*$, then at all $t \in (t_1^*, t_2^*)$, n_t is pinned down by the ODE in (2). Second, Lemma A.7 characterizes t_1^* and t_2^* in terms of the evolution of $(p_t, \lambda N_t)$. Given these steps, it is easy to see that if an equilibrium exists, it is unique and takes the form in (2). Finally, to verify equilibrium existence, Lemma A.8 shows that the adoption flow implied by (2) is feasible.

A.3.1 *Characterization of adoption between t_1^* and t_2^**

Lemma A.6. *Suppose (n_t) is an equilibrium with associated no-news posterior (p_t), and cutoff times t_1^* and t_2^* given by (7). Suppose $t_1^* < t_2^*$. Then at all times $t \in (t_1^*, t_2^*)$,*

$$n_t = \frac{r(2p_t - 1)}{\lambda(1 - p_t)} - \frac{\varepsilon}{\lambda}.$$

Proof. By definition of t_1^* and t_2^*, and Lemma 1, we have $2p_t - 1 = W_t$ at all $t \in (t_1^*, t_2^*)$ (see footnote 15). Because p_t is weakly increasing, this implies that p_t and W_t are differentiable at almost all $t \in (t_1^*, t_2^*)$ (with respect to Lebesgue measure).

For all $t \in (t_1^*, t_2^*)$, we can write

$$\begin{aligned} W_t &= e^{-r(t_2^*-t)}\big(p_t + (1-p_t)e^{-\int_t^{t_2^*}(\varepsilon+\lambda n_s)\,ds}\big)(2p_{t_2^*}-1) \\ &= e^{-r(t_2^*-t)}\big(p_t - (1-p_t)e^{-\int_t^{t_2^*}(\varepsilon+\lambda n_s)\,ds}\big). \end{aligned} \tag{8}$$

Here, the first equality holds because indifference throughout (t, t_2^*) ensures that consumers are willing to delay until t_2^*, where the continuation value absent breakdowns is $W_{t_2^*} = 2p_{t_2^*} - 1$. The second equality holds by (1).

Consider any $t \in (t_1^*, t_2^*)$ at which W_t and p_t are differentiable. Combining $\dot{p}_t = p_t(1 - p_t)(\varepsilon + \lambda n_t)$ with (8), we obtain

$$\dot{W}_t = \big(r + (\varepsilon + \lambda n_t)(1 - p_t)\big)W_t. \tag{9}$$

Furthermore, because $W_t = 2p_t - 1$ for all $t \in (t_1^*, t_2^*)$, we must have

$$\dot{W}_t = 2\dot{p}_t = 2p_t(1 - p_t)(\varepsilon + \lambda n_t). \tag{10}$$

Combining (9), (10), and the fact that $W_t = 2p_t - 1$ then yields

$$n_t = \frac{r(2p_t - 1)}{\lambda(1 - p_t)} - \frac{\varepsilon}{\lambda}$$

for almost all $t \in (t_1^*, t_2^*)$. By continuity of p_t and right-continuity of n_t, the identity must then hold for all $t \in (t_1^*, t_2^*)$. □

The following result is an immediate corollary of Lemma A.6.

Corollary A.1. *The posterior at all $t \in (t_1^*, t_2^*)$ evolves according to the ODE $\dot{p}_t = rp_t(2p_t - 1)$. Given an initial condition $p = p_{t_1^*}$, this ODE admits the unique solution*

$$p_t = \frac{p_{t_1^*}}{2p_{t_1^*} - e^{r(t-t_1^*)}(2p_{t_1^*} - 1)}.$$

A.3.2 *Characterization of cutoff times*

Lemma A.7. *Let (n_t) be an equilibrium with corresponding no-news posterior (p_t) and cutoff times t_1^* and t_2^* as defined by (7), and let $(\Lambda_t) := (\lambda N_t)$ describe the evolution of the economy's potential for social learning. Then*

(i) $t_2^* = \inf\{t \geq 0 : \Lambda_t < \Lambda^*(p_t)\}$

(ii) $t_1^* = \min\{t_2^*, \sup\{t \geq 0 : p_t < p^*\}\}$.[41]

[41] By convention, if $\{t \geq 0 : p_t < p^* = \frac{1}{2}\} = \emptyset$, then $\sup\{t \geq 0 : p_t < p^* = \frac{1}{2}\} := 0$.

Proof. We first prove (i) and (ii) under the assumption that either $\varepsilon > 0$ or $p_0 > \frac{1}{2}$. Note that, in this case, Lemma A.4 implies that $\lim_{t\to\infty} 2p_t - 1 > \lim_{t\to\infty} W_t$, whence $t_2^* < \infty$. Moreover, p_t is strictly increasing for all $t > 0$.

For (i), note that by definition of $t_2^* := \sup\{t \geq 0 : n_t < \rho N_t\}$, we have that $2p_t - 1 \geq W_t = G(p_t, \Lambda_t)$ for all $t \geq t_2^*$. This implies that $\Lambda_{t_2^*} \leq \Lambda^*(p_{t_2^*})$. Moreover, for all $t > t_2^*$, $\Lambda_t < \Lambda_{t_2^*}$ and $p_t > p_{t_2^*}$, so since Λ^* is nondecreasing, we have $\Lambda_t < \Lambda^*(p_t)$. Suppose that $0 < t_2^*$. Then by continuity we must have $2p_{t_2^*} - 1 = W_{t_2^*} = G(p_{t_2^*}, \Lambda_{t_2^*})$ and so $\Lambda_{t_2^*} = \Lambda^*(p_{t_2^*})$. Since for all $s < t_2^*$, we have $\Lambda_s \geq \Lambda_{t_2^*}$ and $p_s < p_{t_2^*}$, this implies $\Lambda_s \geq \Lambda^*(p_s)$. This establishes (i).

For (ii), it suffices to prove the following three claims:

(a) If $t_2^* > 0$, then $p_{t_2^*} < p^\sharp$.

(b) If $t_1^* > 0$, then $p_{t_1^*} \leq \overline{p}$.

(c) If $t_1^* < t_2^*$, then $p_{t_1^*} \geq \overline{p}$.

Indeed, given (a) and (b), if $0 < t_1^* = t_2^*$, then $p_{t_1^*} \leq p^*$. Given (a)–(c), if $0 < t_1^* < t_2^*$, then $p_{t_1^*} = \overline{p} = p^*$. If $0 = t_1^* < t_2^*$, then (c) implies that $p_0 \geq \overline{p} = p^*$. In all three cases (ii) readily follows. Finally, if $0 = t_1^* = t_2^*$, then there is nothing to prove.

For claim (a), recall from the above that if $t_2^* > 0$, then $\Lambda_{t_2^*} = \Lambda^*(p_{t_2^*})$, whence $p_{t_2^*} < p^\sharp$ because $\Lambda^*(p^\sharp) = \infty$.

For claim (b), note that if $t_1^* > 0$, then $n_t = 0$ for all $t < t_1^*$. Then for all $t < t_1^*$, $W_t \geq 2p_t - 1$ and by the proof of Lemma A.3, $\dot{W}_t = (r + (1 - p_t)\varepsilon)W_t$. Since $W_{t_1^*} = 2p_{t_1^*} - 1$, we must then have

$$\begin{aligned} 0 &\geq \lim_{\tau\uparrow t_1^*} \dot{W}_\tau - 2\dot{p}_\tau \\ &= \big(r + (1 - p_{t_1^*})\varepsilon\big)(2p_{t_1^*} - 1) - 2p_{t_1^*}(1 - p_{t_1^*})\varepsilon = r(2p_{t_1^*} - 1) - \varepsilon(1 - p_{t_1^*}), \end{aligned}$$

which implies that

$$p_{t_1^*} \leq \frac{\varepsilon + r}{\varepsilon + 2r} =: \overline{p}.$$

Finally, for claim (c), suppose $t_1^* < t_2^*$. Lemma A.6 implies that for all $\tau \in (t_1^*, t_2^*)$,

$$0 \leq n_\tau = \frac{r(2p_\tau - 1)}{\lambda(1 - p_\tau)} - \frac{\varepsilon}{\lambda}.$$

This implies that for all $\tau \in (t_1^*, t_2^*)$,

$$p_\tau \geq \frac{\varepsilon + r}{\varepsilon + 2r} =: \overline{p}$$

and, hence, by continuity, $p_{t_1^*} \geq \overline{p}$ as claimed. This proves the lemma if $\varepsilon > 0$ or $p_0 > \frac{1}{2}$.

Finally, if $\varepsilon = 0$ and $p_0 \leq \frac{1}{2}$, then by Lemma A.5, $n_t = 0$ for all t. Thus, by definition, $t_1^* = t_2^* = \infty$. Moreover, $p_t = p_0 \leq \frac{1}{2}$ and $\Lambda_t = \Lambda_0 > 0$ for all t, so $\inf\{t : \Lambda_t < \Lambda^*(p_t) = 0\} = \sup\{t : p_t < p^* = \frac{1}{2}\} = \infty$, as required. □

From Lemma A.6, it is immediate that if an equilibrium exists, it must take the form of the adoption flow given by (2) in Theorem 1. Moreover, it is easy to see that, given initial parameters, Lemma A.7 and (2) uniquely pin down the times t_1^* and t_2^* as well as the evolution of (p_t) and (n_t) at all times,[42] and that whenever $t_1^* < t_2^* < \infty$, then $2p_t - 1 = W_t$ for all $t \in [t_1^*, t_2^*]$. Provided the adoption flow (n_t) pinned down in this manner is feasible, it is then easy to check that it constitutes an equilibrium.

A.3.3 *Feasibility* It remains to check feasibility, which is nontrivial only at times $t \in (t_1^*, t_2^*)$.

Lemma A.8. *Suppose (n_t) is an adoption flow satisfying (2) in Theorem 1 such that $t_1^* < t_2^*$. Then for all $t \in (t_1^*, t_2^*)$, $n_t \leq \rho N_t$.*

Proof. It suffices to show that

$$\lim_{t \uparrow t_2^*} n_t \leq \rho N_{t_2^*}.$$

This implies the lemma, as $\rho N_t - n_t$ is strictly decreasing in t at all times in (t_1^*, t_2^*).

To see this, suppose by way of contradiction that $\rho N_{t_2^*} < \lim_{t \uparrow t_2^*} n_t$. By continuity this means that there exists some $\nu > 0$ such that $\rho N_t < n_t$ for all $t \in (t_2^* - \nu, t_2^*)$. Note that from the indifference condition at t_2^*, we have that $2p_{t_2^*} - 1 = G(p_{t_2^*}, \lambda N_{t_2^*})$. Furthermore because $\Lambda^*(p_t)$ is increasing in t, $2p_t - 1 < G(p_t, \Lambda_t)$ for all $t < t_2^*$.

Since at all $t \in (t_2^* - \nu, t_2^*)$, $n_t > \rho N_t$, this implies that $W_t > G(p_t, \Lambda_t) > 2p_t - 1$. This is a contradiction since we already checked that the described adoption flow satisfies the condition that $W_t = 2p_t - 1$ for all $t \in (t_1^*, t_2^*)$. □

A.4 *Proof of Lemma 2*

Assume that $p_0 \in (0, p^\sharp)$ and impose Condition 1. Define $\overline{\Lambda}_0 := \max\{\Lambda^*(p_0), \Lambda^*(\overline{p})\}$. Consider any Λ_0 and let $t_i^* := t_i^*(\Lambda_0)$ for $i = 1, 2$ (by the proof of Theorem 1, t_i^* depend on λ, N_0 only through $\Lambda_0 = \lambda N_0$). We show that $t_1^* < t_2^*$ if and only if $\Lambda_0 > \overline{\Lambda}_0$.

Suppose first that $\Lambda_0 > \overline{\Lambda}_0$. By the proof of the first part of Lemma A.7, we must have $t_2^* > 0$ and $\Lambda_{t_2^*} = \Lambda^*(p_{t_2^*})$. If $t_1^* = t_2^* =: t^*$, then by claims (a) and (b) in the proof of Lemma A.7, we must have $p_{t^*} \leq \overline{p}$. Combining these statements, we get

$$\Lambda_{t^*} = \Lambda_0 > \Lambda^*(\overline{p}) \geq \Lambda^*(p_{t^*}) = \Lambda_{t^*},$$

which is a contradiction.

Suppose conversely that $t_1^* < t_2^*$. Then by the proof of Lemma A.7, we have that $\Lambda^*(p_{t_1^*}) < \Lambda_{t_1^*} = \Lambda_0$. That proof also implies that if $0 < t_1^* < t_2^*$, then $p_{t_1^*} = \overline{p} \geq p_0$, and if $0 = t_1^* < t_2^*$, then $p_{t_1^*} = p_0 \geq \overline{p}$. Thus, either way, $\Lambda_0 > \overline{\Lambda}_0$, as claimed.

[42]The main text elaborated on the latter step assuming Condition 1. Without Condition 1, there are two cases: Either (i) $\varepsilon = 0$ and $p_0 \leq \frac{1}{2}$, in which case $n_t \equiv 0$ and $t_1^* = t_2^* = \infty$ (by Lemma A.5), or (ii) $\varepsilon \geq \rho$, in which case, $p^* := \min\{\overline{p}, p^\sharp\} = p^\sharp$, so Lemma A.7 implies that $n_t = 0$ if $\Lambda_t > \Lambda^*(p_t)$ and $n_t = \rho N_t$ if $\Lambda_t \leq \Lambda^*(p_t)$. Thus, $t_1^* = t_2^*$ is the unique first time at which p_t, evolving according to $\dot{p}_t = p_t(1 - p_t)\varepsilon$, satisfies $\Lambda^*(p_t) \geq \Lambda_0$.

A.5 *Proof of Theorem 2*

Suppose learning is via good news. Theorem 2 follows from the following two lemmas.

Lemma A.9. *Let* (n_t) *be an equilibrium with associated cutoff times* $t_1^* := \inf\{t \geq 0 : n_t < \rho N_t\}$ *and* $t_2^* := \sup\{t \geq 0 : n_t > 0\}$. *Then* $t_1^* = t_2^* =: t^*$.

Proof. Suppose for a contradiction that $t_1^* < t_2^*$. As discussed in the text, from the definition of these cutoff times and Lemma 1, we have $2p_t - 1 = W_t$ for all $t \in (t_1^*, t_2^*)$. Then for all $t \in (t_1^*, t_2^*)$ and $\Delta \in (0, t_2^* - t)$, we have

$$W_t = p_t \int_t^{t+\Delta} (\varepsilon + \lambda n_\tau) e^{-\int_t^\tau (\varepsilon + \lambda n_s)\, ds} e^{-r(\tau - t)} \frac{\rho}{\rho + r}\, d\tau$$
$$+ \big((1 - p_t) + p_t e^{-\int_t^{t+\Delta} (\varepsilon + \lambda n_s)\, ds}\big) e^{-r\Delta}(2p_{t+\Delta} - 1).$$

Here, the first term represents a breakthrough arriving at some $\tau \in (t, t + \Delta)$ in which case consumers adopt from then on, yielding a payoff of $e^{-r(\tau - t)} \frac{\rho}{\rho + r}$. The second term represents no breakthrough arriving prior to $t + \Delta$; in this case, consumers' continuation value can be written as $e^{-r\Delta}(2p_{t+\Delta} - 1)$, as indifference throughout $(t, t + \Delta]$ ensures that consumers are willing to delay until $t + \Delta$ and the continuation value at $t + \Delta$ is $W_{t+\Delta} = 2p_{t+\Delta} - 1$.

Note that we must have $p_t \geq \frac{1}{2}$ on (t_1^*, t_2^*), since $2p_t - 1 = W_t$ and $W_t \geq 0$. Moreover, by the definition of t_2^*, there exists $t \in (t_1^*, t_2^*)$ such that $n_t > 0$. By right-continuity of (n_s), we can pick $\Delta \in (0, t_2^* - t)$ sufficiently small such that $n_\tau > 0$ for all $\tau \in (t, t + \Delta)$. Then

$$p_t \int_t^{t+\Delta} (\varepsilon + \lambda n_\tau) e^{-\int_t^\tau (\varepsilon + \lambda n_s)\, ds} e^{-r(\tau - t)} \frac{\rho}{\rho + r}\, d\tau$$
$$< p_t \int_t^{t+\Delta} (\varepsilon + \lambda n_\tau) e^{-\int_t^\tau (\varepsilon + \lambda n_s)\, ds} \frac{\rho}{\rho + r}\, d\tau = p_t\big(1 - e^{-\int_t^{t+\Delta} (\varepsilon + \lambda n_s)\, ds}\big) \frac{\rho}{\rho + r}.$$

This implies that

$$W_t < p_t\big(1 - e^{-\int_t^{t+\Delta} (\varepsilon + \lambda n_s)\, ds}\big) \frac{\rho}{\rho + r} + \big((1 - p_t) + p_t e^{-\int_t^{t+\Delta} (\varepsilon + \lambda n_s)\, ds}\big)(2p_{t+\Delta} - 1)$$
$$\leq p_t\big(1 - e^{-\int_t^{t+\Delta} (\varepsilon + \lambda n_s)\, ds}\big) + \big((1 - p_t) + p_t e^{-\int_t^{t+\Delta} (\varepsilon + \lambda n_s)\, ds}\big)(2p_{t+\Delta} - 1)$$
$$= 2p_t - 1,$$

where the final equality uses Bayesian updating. This contradicts $W_t = 2p_t - 1$. □

Lemma A.10. *Let* (n_t) *be an equilibrium with corresponding cutoff time* $t^* := t_1^* = t_2^*$ *and no-news posterior* (p_t). *Then* $p_t \leq p^s$ *if and only if* $t \geq t^*$, *where*

$$p^s = \frac{(\varepsilon + r)(r + \rho)}{2(\varepsilon + r)(r + \rho) - \varepsilon\rho}.$$

Proof. Define

$$H_t := p_t \int_0^\infty (\varepsilon + \lambda n_{t+\tau}) e^{-(\varepsilon\tau + \int_t^{t+\tau} \lambda n_s\, ds)} \frac{\rho}{r+\rho} e^{-r\tau}\, d\tau.$$

Thus, H_t represents a consumer's expected payoff to waiting at time t given that from t on he adopts only if there has been a breakthrough and given that the population's flow of adoption follows (n_s). By optimality of W_t, we must have $H_t \leq W_t$ for all t. For any posterior $p \in (0, 1)$, let

$$H(p, 0) := p \int_0^\infty \varepsilon e^{-\varepsilon\tau} \frac{\rho}{r+\rho} e^{-r\tau}\, d\tau = p \frac{\varepsilon\rho}{(\varepsilon + r)(r+\rho)}.$$

That is, $H(p, 0)$ represents a consumer's expected payoff to waiting at posterior p, given that he adopts only once there has been a breakthrough and given that breakthroughs are only generated exogenously.

Note that by definition of t^*, $n_t > 0$ if and only if $t < t^*$. This implies that $H(p_t, 0) < H_t$ if $t < t^*$ and $H(p_t, 0) = H_t = W_t$ if $t \geq t^*$; moreover, $2p_t - 1 \geq W_t$ if $t < t^*$ and $2p_t - 1 \leq W_t$ if $t \geq t^*$. Finally, note that $p^s := \frac{(\varepsilon+r)(r+\rho)}{2(\varepsilon+r)(r+\rho)-\varepsilon\rho}$ has the property that $2p - 1 \leq H(p, 0)$ if and only if $p \leq p^s$.

Combining these observations, if $t < t^*$, then $2p_t - 1 \geq W_t \geq H_t > H(p_t, 0)$, so $p_t > p^s$. If $t \geq t^*$, then $2p_t - 1 \leq W_t = H(p_t, 0)$, so $p_t \leq p^s$, as claimed. □

A.6 *Proof of Proposition 1*

Fix r, ρ, ε, p_0. Suppose Λ_0 is such that $t_1^*(\Lambda_0) < t_2^*(\Lambda_0)$. By the proofs of Theorem 1 and Lemma 2, this means that Condition 1 is satisfied, $p_0 < p^\sharp$, and $\Lambda_0 > \overline{\Lambda}_0$, where $\overline{\Lambda}_0 := \max\{\Lambda^*(p_0), \Lambda^*(\overline{p})\}$ as in the proof of Lemma 2. Consider any $\hat{\Lambda}_0 > \Lambda_0$.

A.6.1 *Proof of part (i) (welfare neutrality)*

Write $\Lambda_0^1 := \Lambda_0$ and $\Lambda_0^2 := \hat{\Lambda}_0$, with corresponding cutoff times t_1^i and t_2^i, value to waiting W_t^i, and no-news posteriors p_t^i for $i = 1, 2$ (by the proof of Theorem 1, these quantities depend on λ^i, N_0^i only through Λ_0^i). Since $t_1^1 < t_2^1$ and $\Lambda_0^2 > \Lambda_0^1 > \overline{\Lambda}_0$, Lemma 2 implies $t_1^2 < t_2^2$. Moreover, by the proof of Lemma A.7, we have $\max\{p_0, \overline{p}\} = p^1_{t_1^1} = p^2_{t_1^2}$. Because $n_t^i = 0$ for all $t < t_1^i$ for both $i = 1, 2$, this implies that $t_1^1 = t_1^2 = t_1$. Then $W^2_{t_1} = 2p^2_{t_1} - 1 = 2p^1_{t_1} - 1 = W^1_{t_1}$. Since there is no adoption until t_1, we have $W_0^i = e^{-rt_1} \frac{p_{t_1}}{p_0} W^i_{t_1}$ for $i = 1, 2$, whence $W_0^1 = W_0^2$, as claimed. □

A.6.2 *Proof of part (ii) (non-monotonicity of learning)*

We first prove the following lemma.

Lemma A.11. *Suppose that $\hat{\Lambda}_0 = \hat{\lambda}\hat{N}_0 > \Lambda_0 = \lambda N_0 > \overline{\Lambda}_0$, with corresponding equilibrium flows of adoption $(\hat{n}_t)$ and (n_t). Then*

(i) $t_1^*(\Lambda_0) = t_1^*(\hat{\Lambda}_0)$

(ii) $0 < t_2^*(\Lambda_0) < t_2^*(\hat{\Lambda}_0)$

(iii) for all $t < t_2^*(\Lambda_0)$, $\lambda n_t = \hat{\lambda}\hat{n}_t$.

Proof. For (i), note that by the proof of Lemma A.7, time t_1^* under both Λ_0 and $\hat{\Lambda}_0$ is pinned down by the condition $\max\{p_0, \overline{p}\} = p^{\Lambda_0}_{t_1^*(\Lambda_0)} = p^{\hat{\Lambda}_0}_{t_1^*(\hat{\Lambda}_0)}$. Because up to time t_1^*, learning is purely exogenous under both Λ_0 and $\hat{\Lambda}_0$, this implies $t_1^*(\Lambda_0) = t_1^*(\hat{\Lambda}_0)$.

For (ii) and (iii), note first that by Lemma 2, we have $t_2^*(\hat{\Lambda}_0), t_2^*(\Lambda_0) > 0$. Let $t_2^* = \min\{t_2^*(\hat{\Lambda}_0), t_2^*(\Lambda_0)\}$. Then because $t_1^*(\Lambda_0) = t_1^*(\hat{\Lambda}_0)$, the ODE in Corollary A.1 implies that at all times $t < t_2^*$, we have $p_t^{\Lambda_0} = p_t^{\hat{\Lambda}_0} = p_t$. By Lemma A.6, this implies that for all $t < t_2^*$,

$$\lambda n_t = \hat{\lambda}\hat{n}_t. \tag{11}$$

Note that (11) implies that

$$\Lambda_{t_2^*} = \Lambda_0 - \int_0^{t_2^*} \lambda n_t \, dt < \hat{\Lambda}_0 - \int_0^{t_2^*} \hat{\lambda}\hat{n}_t \, dt = \hat{\Lambda}_{t_2^*}.$$

Because $p^{\Lambda_0}_{t_2^*} = p^{\hat{\Lambda}_0}_{t_2^*}$, Lemma A.7 implies that $t_2^* = t_2^*(\Lambda_0) < t_2^*(\hat{\Lambda}_0)$. From this and (11), it is then immediate that $\lambda n_t = \hat{\lambda}\hat{n}_t$ for all $t < t_2^*(\Lambda_0)$. □

Now we prove part (ii) of Proposition 1. By Lemma A.11, $t^* := t_2^*(\Lambda_0) < t_2^*(\hat{\Lambda}_0)$, $\lambda n_t = \hat{\lambda}\hat{n}_t$, and $p_t^{\Lambda_0} = p_t^{\hat{\Lambda}_0}$ for all $t \leq t^*$, which proves the first claim of part (ii).

For the second claim of part (ii), we note that there exists some $\nu > 0$ such that at all times $t \in (t^*, t^* + \nu)$, we have $p_t^{\Lambda_0} > p_t^{\hat{\Lambda}_0}$. To see this, we prove the following inequality for the equilibrium corresponding to Λ_0:

$$\lim_{t \uparrow t^*} \lambda n_t < \lim_{t \downarrow t^*} \lambda n_t. \tag{12}$$

That is, there is a discontinuity in the equilibrium flow of adoption at time t^*. Indeed, because $n_t = \rho N_t$ for all $t \geq t^*$ and by continuity of N_t, feasibility implies that $\lim_{t \uparrow t^*} \lambda n_t \leq \lim_{t \downarrow t^*} \lambda n_t$. Suppose for a contradiction that $\lim_{t \uparrow t^*} \lambda n_t = \lim_{t \downarrow t^*} \lambda n_t := \lambda n_{t^*}$. Then $\lambda n_{t^*} = \hat{\lambda}\hat{n}_{t^*}$. Moreover, for all $t > t^*$, we have $\lambda n_t = \rho \Lambda_{t^*} e^{-\rho(t-t^*)}$, which is strictly decreasing in t. On the other hand, $\hat{\lambda}\hat{n}_t$ satisfies

$$\hat{\lambda}\hat{n}_t = \begin{cases} \dfrac{r(2\hat{p}_t - 1)}{(1-\hat{p}_t)} - \varepsilon & \text{if } t \in [t^*, t_2^*(\hat{\Lambda}_0)) \\ \rho \Lambda_{t_2^*(\hat{\Lambda}_0)} e^{-\rho(t - t_2^*(\hat{\Lambda}_0))} & \text{if } t \geq t_2^*(\hat{\Lambda}_0). \end{cases}$$

Thus, for $t \in [t^*, t_2^*(\hat{\Lambda}_0))$, $\hat{\lambda}\hat{n}_t$ is strictly increasing in t. This implies that $\hat{\lambda}\hat{n}_t > \lambda n_t$ for all $t \in [t^*, t_2^*(\hat{\Lambda}_0))$. Hence, by (1), $p^{\hat{\Lambda}_0}_{t_2^*(\hat{\Lambda}_0)} > p^{\Lambda_0}_{t_2^*(\hat{\Lambda}_0)}$, which by Lemma A.7 implies

$$\hat{\Lambda}_{t_2^*(\hat{\Lambda}_0)} = \Lambda^*\big(p^{\hat{\Lambda}_0}_{t_2^*(\hat{\Lambda}_0)}\big) > \Lambda^*\big(p^{\Lambda_0}_{t_2^*(\hat{\Lambda}_0)}\big) > \Lambda_{t_2^*(\hat{\Lambda}_0)}.$$

This yields that for all $t \geq t_2^*(\hat{\Lambda}_0)$,

$$\hat{\lambda}\hat{n}_t = \rho e^{-\rho(t - t_2^*(\hat{\Lambda}_0))} \hat{\Lambda}_{t_2^*(\hat{\Lambda}_0)} > \rho e^{-\rho(t - t_2^*(\hat{\Lambda}_0))} \Lambda_{t_2^*(\hat{\Lambda}_0)} = \lambda n_t.$$

Thus, $\hat{\lambda}\hat{n}_t > \lambda n_t$ for all $t > t^*$ and, hence, $p_t^{\hat{\Lambda}_0} > p_t^{\Lambda_0}$ for all $t > t^*$. This implies $W_{t^*}^{\hat{\Lambda}_0} > W_{t^*}^{\Lambda_0}$, which is a contradiction, because we have

$$W_{t^*}^{\hat{\Lambda}_0} = 2p_{t^*}^{\hat{\Lambda}_0} - 1 = 2p_{t^*}^{\Lambda_0} - 1 = W_{t^*}^{\Lambda_0}.$$

This proves that $\lim_{t\uparrow t^*} \lambda n_t < \lim_{t\downarrow t^*} \lambda n_t$. Hence,

$$\lim_{t\downarrow t^*} \hat{\lambda}\hat{n}_t = \lim_{t\uparrow t^*} \hat{\lambda}\hat{n}_t = \lim_{t\uparrow t^*} \lambda n_t < \lim_{t\downarrow t^*} \lambda n_t.$$

Thus, there exists some $\nu > 0$ such that $\hat{\lambda}\hat{n}_t < \lambda n_t$ for all $t \in [t^*, t^* + \nu)$. Together with the fact that $p_{t^*}^{\Lambda_0} = p_{t^*}^{\hat{\Lambda}_0}$, this implies that $p_t^{\Lambda_0} > p_t^{\hat{\Lambda}_0}$ for all $t \in (t^*, t^* + \nu)$, proving the second claim.

Finally, for the third claim of part (ii), observe first that there exists some $t > t^*$ such that $p_t^{\Lambda_0} = p_t^{\hat{\Lambda}_0}$. If not, then by continuity of beliefs, $p_t^{\Lambda_0} > p_t^{\hat{\Lambda}_0}$ for all $t > t^*$ and we have $W_{t^*}^{\hat{\Lambda}_0} < W_{t^*}^{\Lambda_0}$, again contradicting $W_{t^*}^{\hat{\Lambda}_0} = W_{t^*}^{\Lambda_0} = 2p_{t^*} - 1$. Then $\bar{t} := \sup\{s \in (t^*, t) : p_s^{\Lambda_0} > p_s^{\hat{\Lambda}_0}\}$ exists, with $\bar{t} > t^*$ by the second claim. Further, by continuity, $p_{\bar{t}}^{\Lambda_0} = p_{\bar{t}}^{\hat{\Lambda}_0}$, which implies $\int_0^{\bar{t}} \lambda n_s \, ds = \int_0^{\bar{t}} \hat{\lambda}\hat{n}_s \, ds$. This yields $\Lambda_{\bar{t}} < \hat{\Lambda}_{\bar{t}}$, which implies that $\hat{\lambda}\hat{n}_t > \lambda n_t$ for all $t > \bar{t}$. Indeed, if $\bar{t} \geq t_2^*(\hat{\Lambda}_0)$, this is obvious. If $\bar{t} \in (t^*, t_2^*(\hat{\Lambda}_0))$, then we must have $\lambda n_s < \hat{\lambda}\hat{n}_s$ for some $s < \bar{t}$, which implies that $\lambda n_{s'} < \hat{\lambda}\hat{n}_{s'}$ for all $s' \in (s, t_2^*(\hat{\Lambda}_0))$, because N is strictly decreasing and $\hat{n}$ is strictly increasing on this domain. To see that we also have $\lambda n_{s'} < \hat{\lambda}\hat{n}_{s'}$ for all $s' \geq t_2^*(\hat{\Lambda}_0)$, note that from the above, $p_{t_2^*(\hat{\Lambda}_0)}^{\hat{\Lambda}_0} > p_{t_2^*(\hat{\Lambda}_0)}^{\Lambda_0}$, which as above implies that

$$\hat{\Lambda}_{t_2^*(\hat{\Lambda}_0)} = \Lambda^*\big(p_{t_2^*(\hat{\Lambda}_0)}^{\hat{\Lambda}_0}\big) > \Lambda^*\big(p_{t_2^*(\hat{\Lambda}_0)}^{\Lambda_0}\big) > \Lambda_{t_2^*(\hat{\Lambda}_0)}.$$

Hence, $\hat{\lambda}\hat{n}_t > \lambda n_t$ for all $t > \bar{t}$. Thus, in either case, $p_t^{\hat{\Lambda}_0} > p_t^{\Lambda_0}$ for all $t > \bar{t}$. □

A.6.3 *Proof of part (iii) (slowdown of adoption)* **Adoption of Good Products.** By Lemma A.11, $t_1^*(\Lambda_0) = t_1^*(\hat{\Lambda}_0) =: t_1^*$ and $\lambda n_t = \hat{\lambda}\hat{n}_t$ for all $t \in (t_1^*, t^*)$, where $t^* := t_2^*(\Lambda_0)$. Then for all $t < t^*$,

$$\frac{n_t}{N_0} = \frac{\lambda n_t}{\Lambda_0} = \frac{\hat{\lambda}\hat{n}_t}{\Lambda_0} \geq \frac{\hat{\lambda}\hat{n}_t}{\hat{\Lambda}_0} = \frac{\hat{n}_t}{\hat{N}_0},$$

with strict inequality for all $t \in (t_1^*, t^*)$. Therefore, $A_t(\Lambda_0, G) \geq A_t(\hat{\Lambda}_0, G)$ for all $t < t^*$, with strict inequality for all $t \in (t_1^*, t^*)$.

Finally note that for all $t \geq t^*$, $n_t = \rho N_t$ and so

$$A_t(\Lambda_0, G) = A_{t^*}(\Lambda_0, G) + \big(1 - e^{-\rho(t-t^*)}\big)\big(1 - A_{t^*}(\Lambda_0, G)\big)$$
$$A_t(\hat{\Lambda}_0, G) \leq A_{t^*}(\hat{\Lambda}_0, G) + \big(1 - e^{-\rho(t-t^*)}\big)\big(1 - A_{t^*}(\hat{\Lambda}_0, G)\big),$$

where the second inequality follows from feasibility. Because $A_{t^*}(\Lambda_0, G) > A_{t^*}(\hat{\Lambda}_0, G)$, it follows that $A_t(\Lambda_0, G) > A_t(\hat{\Lambda}_0, G)$ for all $t > t_1^*$, as claimed.

Adoption of Bad Products. Recall that $A_t(\lambda, N_0, B)$ denotes the *expected* proportion of adopters at time t conditional on $\theta = B$. That is, letting (n_t) denote the associated equilibrium, we have

$$A_t(\lambda, N_0, B) := \int_0^t (\varepsilon + \lambda n_\tau) e^{-\int_0^\tau (\varepsilon+\lambda n_s)\, ds} \left(\int_0^\tau \frac{n_s}{N_0}\, ds \right) d\tau + e^{-\int_0^t (\varepsilon+\lambda n_s)\, ds} \int_0^t \frac{n_s}{N_0}\, ds$$

$$= \int_0^t \frac{n_\tau}{N_0} e^{-\int_0^\tau (\varepsilon+\lambda n_s)\, ds}\, d\tau,$$

where the final equality follows from integration by parts. Moreover, from the Markovian description of equilibrium in the proof of Theorem 1, it is easy to see that this expression depends on λ and N_0 only through $\Lambda_0 = \lambda N_0$, so we can denote it by $A_t(\Lambda_0, B)$. Then we can assume without loss of generality that Λ_0 and $\hat{\Lambda}_0$ are of the form $\Lambda_0 = \lambda N_0$ and $\hat{\Lambda}_0 = \hat{\lambda} N_0$, i.e., that the two environments have the same population size N_0.

Let (n_t) and $(\hat{n}_t)$ be the equilibrium under λ and $\hat{\lambda}$, respectively. Given an arbitrary strictly positive adoption flow (m_s) and $t > 0$, note that the map

$$\lambda \mapsto \int_0^t m_\tau e^{-\int_0^\tau (\varepsilon+\lambda m_s)\, ds}\, d\tau$$

is strictly decreasing in λ. Since $\hat{\Lambda}_0 > \Lambda_0 > \overline{\Lambda}_0$, we have $t_1^*(\Lambda_0) = t_1^*(\hat{\Lambda}_0) =: t_1^*$, and so we get that for all $t > 0$,

$$\int_0^t n_\tau e^{-\int_0^\tau (\varepsilon+\lambda n_s)\, ds}\, d\tau \geq \int_0^t n_\tau e^{-\int_0^\tau (\varepsilon+\hat{\lambda} n_s)\, ds}\, d\tau, \tag{13}$$

with strict inequality for all $t > t_1^*$. We now show that

$$\int_0^t n_\tau e^{-\int_0^\tau (\varepsilon+\hat{\lambda} n_s)\, ds}\, d\tau \geq \int_0^t \hat{n}_\tau e^{-\int_0^\tau (\varepsilon+\hat{\lambda} \hat{n}_s)\, ds}\, d\tau.$$

Together with (13), this implies the desired conclusion that $A_t(\hat{\lambda} N_0, B) \leq A_t(\lambda N_0, B)$ for all $t > 0$, with strict inequality for all $t > t_1^*$.

To prove this, suppose for a contradiction that there exists some $t > 0$ such that

$$\int_0^t n_\tau e^{-\int_0^\tau (\varepsilon+\hat{\lambda} n_s)\, ds}\, d\tau < \int_0^t \hat{n}_\tau e^{-\int_0^\tau (\varepsilon+\hat{\lambda} \hat{n}_s)\, ds}\, d\tau. \tag{14}$$

Note that by the above result for *good* products, $N_0 A_\tau(\lambda, G) = \int_0^\tau n_s\, ds \geq \int_0^\tau \hat{n}_s\, ds = N_0 A_\tau(\hat{\lambda}, G)$ for all $\tau \geq 0$ and so, for all $t \geq 0$,

$$\int_0^t \varepsilon e^{-\int_0^\tau (\varepsilon + \hat{\lambda} n_s)\, ds}\, d\tau \leq \int_0^t \varepsilon e^{-\int_0^\tau (\varepsilon + \hat{\lambda} \hat{n}_s)\, ds}\, d\tau. \tag{15}$$

Inequalities (14) and (15) together imply

$$\int_0^t (\varepsilon + \hat{\lambda} n_\tau) e^{-\int_0^\tau (\varepsilon + \hat{\lambda} n_s)\, ds}\, d\tau < \int_0^t (\varepsilon + \hat{\lambda} \hat{n}_\tau) e^{-\int_0^\tau (\varepsilon + \hat{\lambda} \hat{n}_s)\, ds}\, d\tau.$$

This is equivalent to

$$\left(1 - e^{-\int_0^t (\varepsilon + \hat{\lambda} n_s)\, ds}\right) < \left(1 - e^{-\int_0^t (\varepsilon + \hat{\lambda} \hat{n}_s)\, ds}\right),$$

which contradicts $\int_0^t n_s\, ds \geq \int_0^t \hat{n}_s\, ds$. □

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