

# On the neutrality of socially responsible investing: The general equilibrium perspective

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This paper investigates the conditions under which socially responsible investment (SRI) is neutral from the viewpoint of general equilibrium theory. Three conditions are jointly sufficient for neutrality of SRI. First, the financial market is complete and SRI does not compromise the spanning opportunities it provides. Second, consumers' rankings of consumption bundles are unaffected by their asset holdings. Third, firms maximize shareholder value. Under an additional assumption that is satisfied, e.g., if SRI takes the form of negative screening, the taxes and transfers needed to implement a Pareto-optimal allocation are the same as in the absence of SRI. SRI is neutral despite financial market incompleteness if there are perfect substitutes for targeted stocks.

**KEYWORDS.** Socially responsible investing, general equilibrium, market incompleteness.

**JEL CLASSIFICATION.** D51, D52, D53, G13, M14.

## 1. INTRODUCTION

Social responsibility has become an important factor in asset allocation. According to the Global Sustainable Investment Alliance (2020, p. 9), the volume of socially responsible investment (SRI) assets was \$35.3 trillion at the beginning of 2020, up more than 50 percent over the preceding 4 years, or 35.9 percent of total assets under professional management. It seems natural to suppose that such multi-trillion dollar investments have the potential to move markets and the real economy. Evidently, this view is underlying the European Union's sustainable finance strategy based on its taxonomy for sustainable activities.<sup>1</sup> Yet, even though there is a sizable theoretical and empirical literature (reviewed below), the question of what are the precise theoretical conditions under which SRI affects asset returns and corporate behavior has not yet been sorted out. The objective of the present paper is to answer this question, using the cornerstone models of resource allocation and consumption-based asset pricing, viz. the Arrow–Debreu (AD) model of general equilibrium with production, time, and uncertainty and the theory of general equilibrium with incomplete markets (GEI).

One important determinant of whether SRI is neutral or not is completeness of the financial market. The standard notion of market completeness entails that the financial

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<sup>1</sup>See [https://ec.europa.eu/info/business-economy-euro/banking-and-finance/sustainable-finance/overview-sustainable-finance\\_en](https://ec.europa.eu/info/business-economy-euro/banking-and-finance/sustainable-finance/overview-sustainable-finance_en).

assets span the state space. Consider the stronger assumption that for each consumer, the assets she does not care about for SRI reasons are sufficient for spanning. Assume further that SRI does not affect consumers' rankings of different consumption vectors, that firms maximize the value of the cash flow they generate for the initial shareholders, and that not all consumers are socially responsible. These four conditions are jointly sufficient for neutrality. SRI is neutral despite market incompleteness if for each firm that is the object of SRI considerations, there is another firm with perfectly correlated outputs that is not the object of SRI considerations, which also ensures replicability of the payoffs of firms targeted by SRI.

With market completeness SRI does not affect socially responsible agents' consumption possibilities at unchanged prices. Separability implies that they choose the same consumption bundle as in the absence of SRI. Similarly, despite incompleteness, SRI does not affect individuals' consumption choices in the perfectly correlated productivity shocks case. Consumers without SRI motives act as counterparties for the desired asset trades. Since the consumers' aggregate supply of funds to the firms is unchanged, shareholder value maximization gives rise to the same corporate investments.

SRI is effective if the financial market is incomplete and the payoffs of stocks targeted by SRI cannot be replicated using other assets, if the same factors that affect consumers' investment decisions also have a direct impact on their preferences over different consumption bundles (cf. Beltratti (2005)), or if pro-social corporate action is initiated by existing shareholders at the expense of shareholder value (see Kitzmüller and Shimshack (2012)).

We also investigate the conditions under which Pareto-optimal allocations can be implemented as parts of market equilibria in the presence of SRI. Implementation is feasible in the complete markets case if it is feasible with contingent commodity markets and SRI satisfies a positive responsiveness condition, which is satisfied, e.g., if the utility or disutility consumers obtain from holding a firm's assets depends on the industry the firm belongs to but not on its scale (as, e.g., in the case of negative screening). The required transfers and taxes are the same as in the contingent commodity markets economy. Thus, the classical "division of labor between government and firms" (Kitzmüller and Shimshack (2012, p. 55)) in achieving economic efficiency is left unaffected by SRI if markets are complete.

Our paper contributes to the theoretical literature on the real effects of SRI. Davidson, Worrell, and El-Jelly (1995), Angel and Rivoli (1997), Statman (2000), Johnsen (2003), and Haigh and Hazelton (2004) provide early verbal and partial equilibrium discussions. In a seminal contribution, Heinkel, Kraus, and Zechner (2001) analyze an equilibrium model in which the threat of a shareholder boycott by sufficiently many pro-social investors induces firms to "go green." Related contributions include Beltratti (2005), Mackey, Mackey, and Barney (2007), Gollier and Pouget (2014), Dam and Scholtens (2015), Vanwalleghem (2017), and Albuquerque, Koskinen, and Zhang (2019). A series of recent papers by Luo and Balvers (2017), Zerbib (2020), Pedersen, Fitzgibbons, and Pomorski (2021), and Pastor, Stambaugh, and Taylor (2021) uses the capital asset pricing model (CAPM) to characterize the impact of SRI on stock returns.

The present paper deviates from the existing literature on SRI by taking the AD–GEI framework with production as the starting point. This approach allows a detailed analysis of the precise determinants of the efficacy of SRI. It highlights the role played by financial market completeness or, alternatively, by the possibility of replicating the pay-offs on stocks that are the object of SRI concerns with other stocks. It brings to the fore the role of separability of consumption choices from portfolio decisions, connects consumers' financial decisions to firms' investment decisions, and sheds light on the role of financial arbitrage in neutralizing the impact of SRI on consumption. It clarifies the conditions for unanimity about shareholder value maximization and allows investigation of the conditions under which Pareto-optimal allocations can be implemented as parts of market equilibria. It provides a consumption-based asset pricing explanation of real effects of SRI using stochastic discount factors. It contains the CAPM with incomplete markets as a special case.

Viewed from a different angle, the paper contributes to the analysis of nonstandard preferences in general equilibrium. Our neutrality result and the analysis of implementation of Pareto-optimal allocations with complete markets are formally similar to the Dufwenberg, Heidhues, Kirchsteiger, Riedel, and Sobel (2011, henceforth: DHKRS) equilibrium equivalence theorem and their version of the Second Welfare Theorem, respectively, for other-regarding preferences (ORP), and the separability condition is borrowed from them. The investigation of SRI in general equilibrium raises many new issues, however, concerning spanning, investment, arbitrage, and shareholder unanimity.

There is a large empirical literature on SRI that analyzes the relative performance of SRI compared to standard investment strategies and the efficacy of SRI in promoting pro-social firm behavior. Some studies find positive abnormal returns (or a higher cost of capital) for socially responsible stocks (e.g., Gompers, Ishii, and Metrick (2003), Kempf and Osthoff (2007), Edmans (2011), Baker, Bergstresser, Serafeim, and Wurgler (2020)). Others report a negative return premium for environment-friendly or pro-ESG (environmental, social, and governance) stocks (Renneboog, Ter Horst, and Zhang (2008), Pedersen, Fitzgibbons, and Pomorski (2021)) and positive premiums for “sin stocks” (e.g., Hong and Kacperczyk (2007), Luo and Balvers (2017), Zerbib (2020)) or firms excluded from SRI portfolios due to their ESG performance (e.g., El Ghouli, Guedhami, Kwok, and Mishra (2011), Chava (2014)). Still others do not find a significant impact of SRI orientation on returns (e.g., Galema, Plantinga, and Scholtens (2008)). According to Junkus and Berry's (2015, p. 1183) survey of the impact of SRI on returns, “[M]ost studies found that there is no statistically significant difference in the performance of SR [socially responsible] and conventional mutual funds”. Similarly, while some authors argue that screening can provide incentives for firms to behave socially responsibly (e.g., Aslaksen and Synnøstvedt (2003)), others are skeptical (e.g., Haigh and Hazelton (2004)). A prominent example of neutrality of SRI is Teoh, Welch, and Wazzan's (1999) finding that the events leading to the passage of the Anti-Apartheid Act of 1986 in the United States, which included the prohibition of new loans and investments, did not hurt the South African stock market. Wagemans, van Koppen, and Mol (2013, p. 246) conclude from their survey of existing studies of the impact of SRI on corporate behavior that “SRI is

currently not a major driver for social and environmental change.” Our neutrality results help explain why the empirical evidence is less conclusive than one might expect in view of the size of the SRI industry. By identifying conditions that determine the efficacy of SRI, our analysis generates new testable hypotheses as to the circumstances under which SRI is or is not related to returns and capital formation.

The paper is organized as follows. Section 2 introduces the model, with a special focus on SRI. Section 3 introduces the relevant notion of market completeness and clarifies the conditions for shareholder value maximization. Sections 4 and 5 analyze the cases of complete and incomplete markets, respectively. Section 6 summarizes the empirical implications of the analysis. Section 7 concludes. Proofs are collected in the Appendix.

## 2. MODEL

The model we consider is the standard AD-GEI model with production, time, uncertainty, financial markets, and externalities augmented to include SRI.

There are two dates, 0 and 1. At date 1, there is a finite number of states, labeled  $s = 1, 2, \dots, S$ . At date 0, there is a single good. At date 1, there are  $L$  different goods, labeled  $l = 1, 2, \dots, L$ .

There are  $J_l$  single-output firms producing good  $l$  labeled  $j = 1, \dots, J_l$ . The  $j$ th firm producing good  $l$  is briefly called  $(j, l)$ . It transforms capital input  $k_{jl} (\geq 0)$  at date 0 into output  $y_{jls} (\geq 0)$  in state  $s$  at date 1 according to the production function  $f_{jls}(k_{jl})$ . Marginal returns are positive and strictly decreasing, so that firms generate a positive cash flow for their initial owners. The vector of firm capital stocks is denoted  $\mathbf{k} = (k_{11}, \dots, k_{j1}, \dots, k_{J_1L})$ . The state dependence of the production function allows for any kind of productivity shocks, a first source of risk in the economy. We allow for externalities that emanate from firms' production processes and harm or benefit consumers. Pro-social firm behavior consists of reductions in the scale of operations that cause negative externalities or expansion of activities with positive externalities beyond what shareholder value maximization indicates. We argue below that the analysis generalizes straightforwardly to a setup with externalities in production, abatement costs, or a choice between technologies that differ with respect to the amount of externalities they cause.

The economy is populated by  $I$  consumers labeled  $i = 1, 2, \dots, I$ . Consumer  $i$ 's consumption vector is  $\mathbf{c}_i = (c_{i0}, c_{i11}, \dots, c_{iLS})$ , where  $c_{i0} (\geq 0)$  is her consumption at date 0 and  $c_{i1s} (\geq 0)$  is her consumption of good  $l$  in state  $s$  at date 1. Consumer  $i$  is endowed with  $y_{i0} (> 0)$  units of the single available good at date 0 (and nothing at date 1). These endowments can be used for date-0 consumption. Alternatively they can be transformed one-to-one into capital supplied to the firms. Consumer  $i$ 's consumption utility is given by the function  $u_i(\mathbf{c}_i, \mathbf{k})$ . Utility  $u_i$  can have the standard expected utility form. Alternatively, the amount of utility drawn from a given level of consumption can differ across states, so that “preference shocks” are a second source of risk in the economy. The dependence on  $\mathbf{k}$  captures the negative and positive externalities emanating from the firm sector. Utility  $u_i$  allows for both local externalities (in which case  $i$ 's utility

is only affected by a subset of geographically nearby producers  $(j, l)$  and global externalities like climate change (in which case the capital stocks of all firms that emit  $\text{CO}_2$  affect  $u_i$ ). It captures the impact of externalities on  $i$  at both dates and allows for a differential impact of production on  $i$ 's utility in different states  $s$  at date 1, a third source of risk. For instance, if the externalities emanating from firm  $(j, l)$ 's productive activity are particularly harmful for  $i$  in a subset  $\mathcal{S}$  of states (e.g., when they interact with meteorological factors to cause natural disasters), her consumption utility  $u_i$  may depend strongly negatively on  $f_{jls}(k_{jl})$  for  $s \in \mathcal{S}$  but not for  $s \notin \mathcal{S}$ . The utility function  $u_i$  may also reflect  $i$ 's attitudes toward the environmental or other external consequences of her consumption  $c_i$ .

SRI is introduced by assuming that, as argued by Fama and French (2007, p. 675), some consumers “get direct utility from their holdings of some assets, above and beyond the utility from general consumption that the payoffs on the assets provide.” Riedl and Smeets (2017, p. 2508), among others, provide empirical support for the assumption that “social preferences are indeed an important determinant of investment decisions.” Anticipating the existence of stock and bond markets, let  $\theta_{ijl}$  denote  $i$ 's ownership share in firm  $(j, l)$  and let  $a_{ijl}$  denote her holdings of bonds issued by  $(j, l)$ . The respective vectors of asset holdings are denoted  $\theta_i = (\theta_{i11}, \dots, \theta_{iJLL})$  and  $a_i = (a_{i11}, \dots, a_{iJLL})$ . The term  $v_{jl}$  denotes the market capitalization of firm  $(j, l)$ , i.e., the total value at which the firm's shares are traded, and  $v = (v_{11}, \dots, v_{JLL})$ . *Socially responsible (SR) consumers'* overall utility function  $U_i$  depends on consumption utility  $u_i(c_i, k)$  as well as on  $\theta_i, a_i, k$ , and  $v$ , i.e., it satisfies the following notion of separability:

**DEFINITION 1.** Consumer  $i$ 's utility function satisfies *separability* if it is a function  $U_i(u_i(c_i, k), \theta_i, a_i, k, v)$  of her consumption utility  $u_i(c_i, k)$ , and of  $\theta_i, a_i, k$ , and  $v$ .

Given separability,  $i$ 's ranking of any set of consumption vectors  $c_i$  is independent of her asset holdings. As a consequence, SRI does not have an impact on her demands for goods if it leaves the set of consumption vectors she can afford unchanged (see Lemma 3 below). The analogous separability condition is used by DHKRS (2011, Section 2) in their analysis of ORP in general equilibrium (nonseparability is discussed in Section 4.3). Consumers whose overall utility  $U_i$  coincides with their consumption utility  $u_i(c_i, k)$  are called *neutral consumers*.

The specification of  $U_i$  encompasses several different types of SRI. Consumer  $i$ 's utility can depend on the number of shares she holds in firm  $(j, l)$  (i.e., on  $\theta_{ijl}$  but not on  $v_{jl}$ ) or on the value of her shareholdings  $\theta_{ijl}v_{jl}$ . The values  $\theta_{ijl}$  or  $\theta_{ijl}v_{jl}$  can interact with the firm's capital stock  $k_{jl}$ . For instance,  $U_i$  can depend negatively on  $\theta_{ijl}f_{jls}(k_{jl})$  if  $i$  is concerned about the scale of negative externalities emanating from  $(j, l)$ 's production activities. In the opposite case, when  $U_i$  does not depend directly on  $k$ , so that the scale of the firms does not matter for consumers, we say that SRI is *classification-based*. An example is negative screening, caused by a large disutility in case of  $\theta_{ijl} \neq 0$  irrespective of  $k_{jl}$ . The specification of  $U_i$  allows for a state-contingent assessment of  $i$ 's investments. For instance,  $U_i$  can depend strongly negatively on  $\theta_{ijl}f_{jls}(k_{jl})$  and  $a_{ijl}f_{jls}(k_{jl})$  for a subset  $\mathcal{S}$  of the state space if the externalities caused by the firm are particularly severe

in states  $s \in \mathcal{S}$ . An SR consumer's overall utility does not necessarily depend on her holdings of assets of all firms.

**DEFINITION 2.** Given  $\theta_i$  and  $a_i$ , let  $\theta'_i$  and  $a'_i$  be obtained by replacing  $\theta_{ijl}$  and  $a_{ijl}$  with  $\theta'_{ijl}$  and  $a'_{ijl}$ , respectively, for some  $(j, l)$ . Consumer  $i$  is said to be *indifferent toward the assets issued by  $(j, l)$*  (or, briefly, *indifferent toward  $(j, l)$* ) if for all  $c_i, \theta_i, a_i, k, v, \theta'_{ijl}$ , and  $a'_{ijl}$ ,

$$U_i(u_i(c_i, k), \theta_i, a_i, k, v) = U_i(u_i(c_i, k), \theta'_i, a'_i, k, v).$$

If  $i$  is not indifferent toward  $(j, l)$ , she is said to *care about* the firm's assets or the firm.

We assume that for each firm  $(j, l)$ , there is a consumer  $i$  who is indifferent toward  $(j, l)$ . A sufficient condition is that there is at least one neutral consumer.

We assume *strong monotonicity* in consumption:  $U_i$  is strictly increasing in its first argument, and  $u_i$  is strictly increasing in each component of  $c_i$ . In the complete markets case, for a solution to a consumer's utility maximization problem to exist, utility must not be monotonically increasing or decreasing in her SRI-motivated asset holdings.

**DEFINITION 3.** The function  $U_i(u_i, \theta_i, a_i, k, v)$  satisfies *satiation in SRI* if for all  $u_i, k$ , and  $v$ , there exists  $(\theta_i, a_i)$  that maximizes  $U_i$ .

Dorflleitner and Nguyen (2016, Table 5, p. 16) provide evidence for satiation: in an online survey for private investors with above average wealth, about 60 percent of the participants who stated that SRI plays a role in their investment decisions answered in the affirmative to the question, "Are your nonmonetary or ethical ambitions satisfied if a certain amount is invested sustainably?" A similar proportion of investors indicated that the optimum proportion of sustainable investing is less than one-half of their total investment.

### 3. MARKETS AND MARKET COMPLETENESS

This section explains which markets are open, introduces the relevant notion of completeness of the financial market, discusses shareholder value maximization, and defines equilibrium.

#### 3.1 Markets

Goods are traded in competitive spot markets. The goods prices at date 0 is denoted  $p_0$  ( $> 0$ ) and the price of good  $l$  in state  $s$  at date 1 as  $p_{ls}$  ( $> 0$ ). The price vector is  $\mathbf{p} = (p_0, p_{11}, \dots, p_{ls}, \dots, p_{LS})$ .

There are three types of asset markets. First, there is a market for safe corporate bonds. Firms sell bonds at date 0 that pay off one unit of income in each state  $s$  at date 1. The number of bonds  $b_{jl}$  ( $\geq 0$ ) issued by  $(j, l)$  is exogenous. To make sure that corporate debt is in fact a safe asset, it is assumed that  $b_{jl} \leq p_{ls} y_{jls}$  in all states  $s$  for all  $(j, l)$  in equilibrium. So corporate bonds issued by different firms are perfect substitutes in

terms of their financial payoffs. As will be seen below, they are traded at the same price, denoted  $R$ , irrespective of how they affect the SR consumers' utilities. Second, there is a competitive stock market in which firms' shares are traded. Shareholdings  $\theta_{ijl}$  entitle  $i$  to a fraction  $\theta_{ijl}$  of  $(j, l)$ 's date-1 revenue net of debt service  $p_{ls}y_{jls} - b_{jl}$ . Consumer  $i$ 's initial ownership shares are denoted  $\bar{\theta}_{ijl} (\geq 0)$ ,  $\sum_{i=1}^I \bar{\theta}_{ijl} = 1$  for all  $(j, l)$ , and  $i$  has to contribute a fraction  $\bar{\theta}_{ijl}$  to  $(j, l)$ 's date-0 capital expenditure net of the amount raised by issuing debt  $p_0k_{jl} - Rb_{jl}$ .<sup>2</sup> Third, there are  $M$  state-contingent securities (briefly called securities in what follows) labeled  $m = 1, \dots, M$ . These securities are in zero net supply and pay off  $x_{ms} (\geq 0)$  units of income in state  $s$ . Options are the most prominent example. One may also think of futures, swaps, insurance contracts, etc. The price of security  $m$  at date 0 is  $q_m$ , and  $\mathbf{q} = (q_1, \dots, q_M)$ . Let  $z_{im}$  denote  $i$ 's demand for security  $m$ . Her vector of security holdings  $\mathbf{z}_i = (z_{i1}, \dots, z_{iM})$  is not an argument of  $i$ 's utility function  $U_i$ : consumers are indifferent toward their holdings of state-contingent securities. There are no short sales constraints.

Consumer  $i$ 's budget constraints are

$$p_0(c_{i0} - y_{i0}) + \sum_{l=1}^L \sum_{j=1}^{J_l} [\bar{\theta}_{ijl}(p_0k_{jl} - Rb_{jl}) + (\theta_{ijl} - \bar{\theta}_{ijl})v_{jl} + Ra_{ijl}] + \sum_{m=1}^M q_m z_{im} \leq 0 \tag{1}$$

$$\sum_{l=1}^L p_{ls}c_{ils} - \sum_{l=1}^L \sum_{j=1}^{J_l} [\theta_{ijl}(p_{ls}y_{jls} - b_{jl}) + a_{ijl}] - \sum_{m=1}^M x_{ms}z_{im} \leq 0, \tag{2}$$

$$s = 1, \dots, S,$$

$i = 1, \dots, I$ . The first constraint says that the sum of  $i$ 's net consumption expenditure, her contributions to firm capital formation, and the cost of her asset portfolio is nonpositive. The latter constraints say that in each state, the payoff on her portfolio is sufficient to cover her consumption expenditure. A consumption vector  $\mathbf{c}_i$  is called *affordable* for  $i$  if there exists a portfolio  $(\boldsymbol{\theta}_i, \mathbf{a}_i, \mathbf{z}_i)$  such that  $\mathbf{c}_i$  and  $(\boldsymbol{\theta}_i, \mathbf{a}_i, \mathbf{z}_i)$  jointly satisfy her budget constraints (1) and (2).

### 3.2 Completeness

Without SRI, the absence of arbitrage opportunities implies the existence of a unique vector of state prices if the financial market is complete, i.e., if the assets' payoff vectors span the state space. The state price for  $s$  is the common cost of all portfolios that pay off one unit of income in  $s$  and nothing in all other states in that case (see Magill and Quinzii (2002), p. 383). SRI motives may in principle prevent SR consumers from exploiting arbitrage opportunities, e.g., if an arbitrage portfolio contains long positions in assets they

<sup>2</sup>The analysis remains unaffected if consumers supply capital to firms proportionally to their post-trade ownership shares  $\theta_{ijl}$ .

consider unacceptable. An alternative, stronger, notion of market completeness, which ensures that arbitrage is also active in the presence of SRI, is that the financial market is complete in the usual sense and SRI does not compromise the spanning opportunities it provides for any consumer.

**DEFINITION 4.** The financial market satisfies *spanning with assets with no social returns* (SWANS) if for each  $i$ , there are  $S$  assets  $i$  is indifferent toward that jointly span the state space.

SWANS ensures that each consumer can achieve (though not necessarily afford) a given consumption vector with given holdings of the stocks and corporate bonds she cares about. A sufficient condition is that there are  $S$  state-contingent securities with linearly independent payoff vectors. But the definition does not rule out that  $i$  also needs stocks issued by firms she is indifferent toward so as to achieve a given payoff profile. We say that the financial market is *complete* if SWANS is satisfied and that it is *incomplete* if not (using the term completeness in a stronger than usual sense).

We assume that the financial market satisfies SWANS to begin with (the case of incomplete markets is analyzed in Section 5). We further assume that prices are such that no consumer can reap an arbitrage profit using only assets she is indifferent toward. Since otherwise there is no solution to her utility maximization problem, this is implied by the definition of equilibrium below. SWANS and no arbitrage jointly imply the existence of a unique vector of state prices.

**LEMMA 1.** Given  $\mathbf{v}$ ,  $R$ ,  $\mathbf{q}$ , and  $p_{ls}y_{jls}$  ( $j = 1, \dots, J_l$ ,  $l = 1, \dots, L$ ,  $s = 1, \dots, S$ ), if there is no arbitrage opportunity for any consumer  $i$  that uses assets  $i$  is indifferent toward only, then there exists a unique and strictly positive vector of state prices  $\mathbf{r} = (r_1, \dots, r_S)$  such that

$$v_{jl} = \sum_{s=1}^S r_s (p_{ls}y_{jls} - b_{jl}), \quad j = 1, \dots, J_l, l = 1, \dots, L \quad (3)$$

$$R = \sum_{s=1}^S r_s \quad (4)$$

$$q_m = \sum_{s=1}^S r_s x_{ms}, \quad m = 1, \dots, M. \quad (5)$$

The state prices are the same as in the absence of SRI. The proof rests on the same arguments that establish the irrelevance of deleting redundant assets in complete markets. As stipulated above, safe corporate bonds are traded at a uniform price.

SWANS implies that consumer  $i$  can achieve any given consumption vector  $c_i$  with a portfolio that includes given quantities of the assets she cares about. From the pricing rules in Lemma 1, fixing the quantities of the assets she cares about does not render an affordable consumption vector unaffordable.



LEMMA 2. Let  $(c_i, \theta_i, a_i, z_i)$  satisfy (1) and (2). Let  $\theta'_{ijl}$  and  $a'_{ijl}$  be given for all  $(j, l)$   $i$  cares about. Then there exist  $\theta'_{ijl}$  and  $a'_{ijl}$  for  $(j, l)$   $i$  does not care about and  $z'_i$  such that  $(c_i, \theta'_i, a'_i, z'_i)$  satisfies (1) and (2).<sup>3</sup>

Jointly with separability, Lemma 2 implies that maximization of overall utility  $U_i$  and maximization of consumption utility  $u_i$  give rise to the same demands for consumption goods.

LEMMA 3. If  $(c_i^*, \theta_i^*, a_i^*, z_i^*)$  maximizes  $U_i(u_i(c_i, k), \theta_i, a_i, k, v)$  subject to (1) and (2), then it maximizes  $u_i(c_i, k)$  subject to (1) and (2). Let  $U_i$  satisfy satiation in SRI. Then, conversely, if  $(c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})$  maximizes  $u_i(c_i, k)$  subject to (1) and (2), then there is  $(\theta_i^*, a_i^*, z_i^*)$  such that  $(c_i^*, \theta_i^*, a_i^*, z_i^*)$  maximizes  $U_i(u_i(c_i, k), \theta_i, a_i, k, v)$  subject to (1) and (2).

The economy thus described, with separability, strong monotonicity, at least one indifferent consumer for each firm, SWANS, and no arbitrage, is denoted  $\mathcal{E}$ . To assess the impact of SRI on returns and the real economy, we compare  $\mathcal{E}$  to the economy without SRI,  $\mathcal{E}_0$  say, in which all consumers are neutral (i.e.,  $U_i(u_i(c_i, k), \theta_i, a_i, k, v) = u_i(c_i, k)$  for all  $i$ ).

### 3.3 Shareholder value maximization

Following the existing literature on SRI, we assume that firms maximize shareholder value (SV), i.e., the value of the cash flow they generate for the incumbent shareholders. This rules out pro-social corporate action initiated by initial shareholders pursuing their own nonfinancial goals (which is discussed in Section 4.3).

Shareholder value is  $v_{jl} - (p_0k_{jl} - Rb_{jl})$ , where  $v_{jl}$  is given by (3). In maximizing SV, the firms hold *competitive expectations*, i.e., they take the state prices  $r$  determined according to Lemma 1 as given (cf. Magill and Quinzii (2002, p. 383)).

From the budget constraints (1) and (2) and the pricing equations (3)–(5),

$$p_0(c_{i0} - y_{i0}) + \sum_{s=1}^S r_s \sum_{l=1}^L p_{ls} c_{ils} - \sum_{l=1}^L \sum_{j=1}^{J_l} \bar{\theta}_{ijl} [v_{jl} - (p_0k_{jl} - Rb_{jl})] \leq 0.$$

An increase in SV expands each initial shareholder’s consumption possibilities. Assuming SV maximization is nonetheless not innocuous in the present setting. For one thing, it means that firms ignore the externalities they cause, captured by the second argument in the consumption utility function  $u_i(c_i, k)$ . This is standard in the theory of externalities (see Baumol and Oates (1988, p. 17)). For another, it implies that firms ignore how their capital stock  $k_{jl}$  and the resulting firm value  $v_{jl}$  directly affect their initial shareholders’ overall utility via the final two arguments of  $U_i(u_i(c_i, k), \theta_i, a_i, k, v)$ . If  $U_i$  depends negatively on  $k_{jl}$  or  $v_{jl}$  for some  $i$  with  $\bar{\theta}_{ijl} > 0$ , then  $i$  might approve a decrease in the firm’s scale below the SV maximizing level.

<sup>3</sup>We adopt the convention that  $\theta'_i = (\theta'_{i11}, \dots, \theta'_{iJL})$ , and similarly for  $a'_i$  and other variables below.

There are two possible justifications for the SV maximization assumption. First, one can assume that indifferent initial shareholders, whose utility is independent of  $\theta_i$ ,  $\mathbf{a}_i$ ,  $\mathbf{k}$ , and  $\mathbf{v}$ ,<sup>4</sup> have the power to establish it as the firm objective. These consumers favor SV maximization because it maximizes their consumption possibilities and they do not have any nonfinancial goals. With majority voting over the firm objective and “one share, one vote,” they have the power to establish SV maximization if the sum of their initial shareholdings  $\bar{\theta}_{ijl}$  exceeds one-half for all firms. Second, one can make additional, restrictive, assumptions on consumers’ utility functions to ensure that the initial shareholders are *unanimous* about SV maximization.<sup>5</sup> A convenient specification is

$$U_i(u_i(\mathbf{c}_i, \mathbf{k}), \theta_i, \mathbf{a}_i, \mathbf{k}, \mathbf{v}) = \hat{U}_i(u_i(\mathbf{c}_i, \mathbf{k}), \theta_{i11}v_{11}^{D_{i11}}, \dots, \theta_{iJL}v_{JL}^{D_{iJL}}, v_M), \tag{6}$$

where  $\hat{D}_{ijl} \in \{0, 1\}$  and  $v_M = \sum_{l=1}^L \sum_{j=1}^{J_l} v_{jl}$ . That is,  $U_i$  is independent of bond holdings and SRI is classification-based. Given total market capitalization  $v_M$ , firm  $(j, l)$ ’s market value  $v_{jl}$  matters either not at all (if  $\hat{D}_{ijl} = 0$ ) or only through the values of  $i$ ’s shareholdings (if  $\hat{D}_{ijl} = 1$ ). The dependence on  $v_M$  allows that investments are normalized by market capitalization (i.e.,  $U_i$  depends on  $\theta_{ijl}v_{jl}/v_M$ ).

LEMMA 4. Let  $U_i = \hat{U}_i$  be given by (6). Taking  $\mathbf{k}$  in  $u_i(\mathbf{c}_i, \mathbf{k})$  and  $v_M$  as given, if  $k_{jl}$  maximizes  $v_{jl} - (p_0k_{jl} - Rb_{jl})$ , then there is no  $k'_{jl}$  that allows  $i$  to achieve a higher maximum utility  $\hat{U}_i$ .

The proof rests on the fact that maximization of consumption possibilities does not interfere with the pursuit of social investment goals if utility is given by (6). For given  $v_{jl}$ , consumer  $i$  can still achieve the values of  $\theta_{ijl}v_{jl}^{D_{ijl}}$  that maximize  $\hat{U}_i$  by choosing the  $\theta_{ijl}$ ’s appropriately.

DEFINITION 5. The vector  $((\mathbf{c}_i, \theta_i, \mathbf{a}_i, \mathbf{z}_i)_{i=1}^I, \mathbf{k}, \mathbf{p}, \mathbf{v}, R, \mathbf{q})$  is an *equilibrium* of  $\mathcal{E}$  if  $(\mathbf{c}_i, \theta_i, \mathbf{a}_i, \mathbf{z}_i)$  maximizes  $U_i$  subject to the budget constraints (1) and (2),  $i = 1, \dots, I$ ,<sup>6</sup>  $k_{jl}$  maximizes  $v_{jl} - (p_0k_{jl} - Rb_{jl})$  with  $v_{jl}$  given by (3) taking  $\mathbf{r}$  as given,  $j = 1, \dots, J_l$ ,  $l = 1, \dots, L$ , the goods markets clear, i.e.,

$$\begin{aligned} \sum_{i=1}^I c_{i0} + \sum_{l=1}^L \sum_{j=1}^{J_l} k_{jl} &= \sum_{i=1}^I y_{i0} \\ \sum_{i=1}^I c_{ils} &= \sum_{j=1}^{J_l} y_{jls}, \quad l = 1, \dots, L, s = 1, \dots, S, \end{aligned}$$

<sup>4</sup>Consumer  $i$  is indifferent toward  $(j, l)$  if  $U_i$  is independent of  $\theta_{ijl}$  and  $a_{ijl}$ , That  $k_{jl}$  and  $v_{jl}$  do not affect  $U_i$  either is an additional assumption.

<sup>5</sup>Evidently, it would be enough that unanimity prevails among a subset of initial shareholders holding a majority of the shares.

<sup>6</sup>The solution is time-consistent: the optimum  $(c_{i1s}, \dots, c_{iLs})$  maximizes  $U_i$  subject to (2) for given  $s$ .

and the asset markets clear, i.e.,

$$\sum_{i=1}^I \theta_{ijl} = 1, \quad \sum_{i=1}^I a_{ijl} = b_{jl}, \quad j = 1, \dots, J, l = 1, \dots, L$$

$$\sum_{i=1}^I z_{im} = 0, \quad m = 1, \dots, M.$$

An equilibrium of  $\mathcal{E}_0$  is the special case with  $U_i(u_i(c_i, \mathbf{k}), \boldsymbol{\theta}_i, \mathbf{a}_i, \mathbf{k}, \mathbf{v}) = u_i(c_i, \mathbf{k})$  for all  $i$ .

#### 4. COMPLETE FINANCIAL MARKETS

This section investigates the determinants of the neutrality or nonneutrality of SRI, and analyzes the implementation of efficient allocations as parts of market equilibria with SWANS.

##### 4.1 Neutrality of SRI

Our first main result is that it does not make a difference if consumers give up SRI.

**THEOREM 1.** *If  $((c_i^*, \boldsymbol{\theta}_i^*, \mathbf{a}_i^*, z_i^*)_{i=1}^I, \mathbf{k}^*, \mathbf{p}^*, \mathbf{v}^*, R^*, \mathbf{q}^*)$  is an equilibrium of  $\mathcal{E}$ , it is also an equilibrium of  $\mathcal{E}_0$ .*

Suppose all goods prices, stock market valuations, security prices and state prices, and the bond price are the same in  $\mathcal{E}_0$  as in  $\mathcal{E}$ . Then firms do not have an incentive to change their outputs  $y_{jls} = f_{jls}(k_{jl}^*)$ . It follows from Lemma 1 that the state prices in fact do not change. Furthermore, the budget constraints are the same in  $\mathcal{E}_0$  as in  $\mathcal{E}$ . From Lemma 3, consumers choose the same consumption vectors  $c_i^*$  and portfolios  $(\boldsymbol{\theta}_i^*, \mathbf{a}_i^*, z_i^*)$ .

Since individual asset holdings are indeterminate, there are a multitude of other equilibria of  $\mathcal{E}_0$  with different portfolios  $(\boldsymbol{\theta}_i, \mathbf{a}_i, z_i)_{i=1}^I$  but with the same allocation  $((c_i^*)_{i=1}^I, \mathbf{k}^*)$  and the same prices  $\mathbf{p}^*, \mathbf{v}^*, R^*$ , and  $\mathbf{q}^*$ .

The converse of Theorem 1, i.e., neutrality of introducing SRI, is not generally true for two reasons. First, SRI may cause non-existence of equilibrium in  $\mathcal{E}$  even if an equilibrium of  $\mathcal{E}_0$  exists. To see this, suppose  $i$ 's marginal utility  $\partial U_i / \partial \theta_{ijl}$  of shares in firm  $(j, l)$  is constant and nonzero. Consider an equilibrium of  $\mathcal{E}$  at which  $i$  consumes  $c_i^*$ . Depending on whether  $\partial U_i / \partial \theta_{ijl}$  is positive or negative,  $i$  can increase her utility by increasing or decreasing  $\theta_{ijl}$ , respectively. From Lemma 2, she can change the holdings of assets she is indifferent toward in such a way that  $c_i^*$  is unaffected. The converse of Theorem 1 thus requires satiation in SRI (see Definition 3).

The second reason why the converse of Theorem 1 is not generally true is that the changes in consumers' portfolios caused by the introduction of SRI may be incompatible with asset market clearing. To see this, suppose there are two equally likely states ( $S = 2$ ) and only one good ( $L = 1$ ). The economy is symmetric with regard to the two

states. The number of firms is even. For  $j$  odd and  $j'$  even, the production functions obey  $f_{j11}(k) = f_{j'12}(k)$  and  $f_{j12}(k) = f_{j'11}(k)$  for all  $k \geq 0$ , where  $f_{j11}(k) > f_{j12}(k)$  for all  $k \geq 0$ . That is, the odd-numbered firms are more productive in state 1 and the even-numbered firms are more productive in state 2. The consumption utility functions are symmetric in that  $u_i(c_{i0}, c, c', \mathbf{k}) = u_i(c_{i0}, c', c, \mathbf{k})$  for all  $c_{i0}, c, c'$ , and  $\mathbf{k}$ . The only security is a safe asset. Consider a symmetric equilibrium of  $\mathcal{E}_0$  at which all firms install the same amount of capital and consumers are fully insured, i.e.,  $c_{i11}^* = c_{i12}^*$  for all  $i$ . Suppose, for social responsibility reasons (a large penalty inherent in  $U_i$ ), the odd-numbered consumers boycott the shares of the odd-numbered firms and the even-numbered consumers boycott the shares of even-numbered firms (that applies to both long positions and short sales). Since for each  $i$ , the non-boycotted stock and the safe asset span the state space, SWANS is satisfied. Additionally, for each firm there is a consumer who is indifferent toward the firm. However, given that  $c_{i11}^* = c_{i12}^*$ , the only way for  $i$  to obtain  $c_i^*$  in  $\mathcal{E}$  is to invest only in the safe asset and not buy shares in the firms she does not boycott anyway. That implies zero demand for each stock, which is incompatible with market clearing. A simple way to overcome this second problem is to assume that there is at least one neutral consumer.

**THEOREM 2.** *Let all  $U_i$  satisfy satiation in SRI and let the set of neutral consumers be non-empty. Then, if  $((c_i^*, \theta_i^*, \mathbf{a}_i^*, \mathbf{z}_i^*)_{i=1}^I, \mathbf{k}^*, \mathbf{p}^*, \mathbf{v}^*, R^*, \mathbf{q}^*)$  is an equilibrium of  $\mathcal{E}_0$ , there is an equilibrium  $((c_i^*, \theta_i^*, \mathbf{a}_i^*, \mathbf{z}_i^*)_{i=1}^I, \mathbf{k}^*, \mathbf{p}^*, \mathbf{v}^*, R^*, \mathbf{q}^*)$  of  $\mathcal{E}$ .*

The proof is similar to the proof of Theorem 1. Given unchanged goods and state prices, firms do not have an incentive to change their outputs  $y_{jls} = f_{jls}(k_{jl}^*)$  in  $\mathcal{E}$ . From Lemma 3, consumers use a utility maximizing portfolio to finance unchanged consumption  $c_i^*$  in  $\mathcal{E}$ . Neutral consumers are ready to adjust their portfolios such that the asset markets clear at unchanged prices.

In the absence of risk (i.e., when  $S = 1$ ), SWANS is satisfied if, for each consumer, there is one asset she does not care about, e.g., a safe security. SRI has no real effects then. Risk is an essential prerequisite for nonneutrality of SRI.

Call  $((c_i)_{i=1}^I, \mathbf{k})$  an *equilibrium allocation* of  $\mathcal{E}$  if there are portfolios  $(\theta_i, \mathbf{a}_i, \mathbf{z}_i)_{i=1}^I$  and prices  $(\mathbf{p}, \mathbf{v}, R, \mathbf{q})$  such that  $((c_i, \theta_i, \mathbf{a}_i, \mathbf{z}_i)_{i=1}^I, \mathbf{k}, \mathbf{p}, \mathbf{v}, R, \mathbf{q})$  is an equilibrium. Let  $\mathcal{A}$  and  $\mathcal{A}_0$  denote the sets of equilibrium allocations of  $\mathcal{E}$  and  $\mathcal{E}_0$ , respectively. From Theorem 1, the set of equilibria of  $\mathcal{E}$  is a subset of the set of the equilibria of  $\mathcal{E}_0$ , so  $\mathcal{A} \subseteq \mathcal{A}_0$ . Under the additional conditions of Theorem 2, the sets of equilibrium allocations with or without SRI are the same:  $\mathcal{A}_0 \subseteq \mathcal{A}$  and, hence,  $\mathcal{A} = \mathcal{A}_0$ .

Theorems 1 and 2 also hold with a choice between alternative modes of production that differ in terms of sustainability, with abatement technologies, and with externalities in production. To see this, suppose each firm  $(j, l)$  has the choice between different modes of production, indexed by  $\kappa_{jl}$  ( $0 \leq \kappa_{jl} \leq 1$ ), where a higher value corresponds to a lower level of negative externalities. Given  $\kappa_{jl}$ , it can reduce negative externalities further by using a fraction  $\nu_{jl}$  ( $0 \leq \nu_{jl} \leq 1$ ) of its capital stock  $k_{jl}$  for abatement. Let  $\boldsymbol{\kappa} = (\kappa_{11}, \dots, \kappa_{JLL})$  and  $\boldsymbol{\nu} = (\nu_{11}, \dots, \nu_{JLL})$ . Firm  $(j, l)$ 's output is given by  $f_{jls}((1 - \nu_{jl})k_{jl}, \kappa_{jl}, \mathbf{k}, \boldsymbol{\kappa}, \boldsymbol{\nu})$ . The function  $f_{jls}$  displays decreasing marginal returns with

respect to its own capital stock used in production  $(1 - \iota_{jl})k_{jl}$ . It is strictly decreasing in its second argument, so that a production mode with lower emissions yields lower production. The remaining three arguments of  $f_{jls}$  capture externalities in production. The components  $k_{jl}$ ,  $\kappa_{jl}$ , and  $\iota_{jl}$  of  $\mathbf{k}$ ,  $\boldsymbol{\kappa}$ , and  $\boldsymbol{\iota}$ , respectively, do not affect  $f_{jls}$ . Consumption utility is  $u_i(c_i, \mathbf{k}, \boldsymbol{\kappa}, \boldsymbol{\iota})$ . SV maximization implies  $\kappa_{jl} = \iota_{jl} = 0$ . The analysis goes through without further modification.

### 4.2 Welfare

Due to externalities, the First Welfare Theorem obviously does not hold in  $\mathcal{E}$  or  $\mathcal{E}_0$ . So, following DHKRS (Section 5), we focus on the question of whether given Pareto-optimal allocations can be implemented as parts of market equilibria using lump-sum transfers. Because of externalities, this also necessitates the use of Pigouvian taxes. We show that any Pareto-optimal allocation that can be implemented in an economy with contingent commodity markets (CCMs) and no financial markets can also be achieved in  $\mathcal{E}$  provided that SRI satisfies a positive responsiveness property that generalizes classification-based SRI. The required lump-sum transfers and Pigouvian taxes are the same in both economies. Thus, given positive responsiveness, SRI does not impinge on the problem of implementing efficient allocations.

Denote the economy with CCMs as  $\tilde{\mathcal{E}}$ . In addition to the date-0 spot markets, there is a CCM for each good  $l$  in each state  $s$  in which the claim to the delivery of one unit of  $l$  conditional on the occurrence of  $s$  is traded at price  $\tilde{p}_{ls}$  at date 0. The price vector is denoted  $\tilde{\mathbf{p}} = (p_0, \tilde{p}_{11}, \dots, \tilde{p}_{LS})$ . There are no financial markets. Consumer  $i$ 's overall utility  $U_i$  coincides with her consumption utility  $u_i$ . Capital supplied to firm  $(j, l)$  at date 0 is taxed at rate  $t_{jl}$ . Consumer  $i$  gets a lump-sum transfer  $T_i$  at date 0. The fiscal budget is balanced:  $\sum_{l=1}^L \sum_{j=1}^{J_l} t_{jl} p_0 k_{jl} = \sum_{i=1}^I T_i$ . Consumer  $i$ 's budget constraint is

$$p_0(c_{i0} - y_{i0}) + \sum_{s=1}^S \sum_{l=1}^L \tilde{p}_{ls} c_{ils} - \sum_{l=1}^L \sum_{j=1}^{J_l} \bar{\theta}_{ijl} \left[ \sum_{s=1}^S \tilde{p}_{ls} y_{jls} - (1 + t_{jl}) p_0 k_{jl} \right] - T_i \leq 0.$$

**DEFINITION 6.** The allocation  $((c_i)_{i=1}^I, \mathbf{k}, \tilde{\mathbf{p}})$  is an *equilibrium* of  $\tilde{\mathcal{E}}$  if each consumer  $i$  maximizes utility  $u_i$  subject to her budget constraint, each firm  $(j, l)$  maximizes SV  $\sum_{s=1}^S \tilde{p}_{ls} f_{jls}(k_{jl}) - (1 + t_{jl}) p_0 k_{jl}$ , and the date-0 spot markets and the CCMs clear.

Under standard convexity assumptions, given any Pareto-optimal allocation  $((c_i)_{i=1}^I, \mathbf{k})$ , there are tax rates  $t_{jl}$  ( $j = 1, \dots, J_l, l = 1, \dots, L$ ), transfers  $T_i$  ( $i = 1, \dots, I$ ), and a price vector  $\tilde{\mathbf{p}}$  such that  $((c_i)_{i=1}^I, \mathbf{k}, \tilde{\mathbf{p}})$  is an equilibrium of  $\tilde{\mathcal{E}}$ . In the differentiable case, letting  $p_0 = 1$ , the Pigouvian taxes are

$$t_{jl} = - \sum_{i=1}^I \frac{\frac{\partial u_i(c_i, \mathbf{k})}{\partial k_{jl}}}{\frac{\partial u_i(c_i, \mathbf{k})}{\partial c_{i0}}},$$

the transfers  $T_i$  make sure that  $i$  can just afford  $c_i$ , and  $\tilde{p}_{ls} = [\partial u_i(c_i, \mathbf{k}) / \partial c_{ls}] / [\partial u_i(c_i, \mathbf{k}) / \partial c_{i0}]$  (cf. Baumol and Oates (1988, Section 4.3)).<sup>7</sup>

In the economy with financial markets  $\mathcal{E}$ , as in the CCM economy, let there be taxes  $t_{jl}$  on capital formation and transfers  $T_i$  to consumers. Consumer  $i$ 's net payment  $\sum_{l=1}^L \sum_{j=1}^{J_l} \tilde{\theta}_{ijl} t_{jl} p_0 k_{jl} - T_i$  shows up as an additional term on the left-hand side of her date-0 budget constraint (1). Let the tax rates and transfers take on the same values in  $\mathcal{E}$  as in  $\tilde{\mathcal{E}}$ , and let the conditions of Theorem 2 be satisfied. Then  $\mathcal{E}$  brings forth the same set of equilibrium allocations as  $\tilde{\mathcal{E}}$ . To see this, consider first the economy without SRI  $\mathcal{E}_0$ . Let  $((c_i, \theta_i, a_i, z_i)_{i=1}^I, \mathbf{k}, \mathbf{p}, \mathbf{v}, R, \mathbf{q})$  be an equilibrium of  $\mathcal{E}_0$  and let  $\tilde{p}_{ls} = r_s p_{ls}$ . Then  $((c_i)_{i=1}^I, \mathbf{k}, \tilde{\mathbf{p}})$  is an equilibrium of  $\tilde{\mathcal{E}}$ . Conversely, if  $((c_i)_{i=1}^I, \mathbf{k}, \tilde{\mathbf{p}})$  is an equilibrium of  $\tilde{\mathcal{E}}$ , then there is an equilibrium  $((c_i, \theta_i, a_i, z_i)_{i=1}^I, \mathbf{k}, \mathbf{p}, \mathbf{v}, R, \mathbf{q})$  of  $\mathcal{E}_0$  with  $\tilde{p}_{ls} = r_s p_{ls}$ . This follows from the fact that the sets of affordable consumption vectors are identical in  $\mathcal{E}_0$  and  $\tilde{\mathcal{E}}$  when  $\tilde{p}_{ls} = r_s p_{ls}$  (see Magill and Quinzii (2002, p. 384)). Call  $((c_i)_{i=1}^I, \mathbf{k})$  an equilibrium allocation of  $\tilde{\mathcal{E}}$  if there is  $\tilde{\mathbf{p}}$  such that  $((c_i)_{i=1}^I, \mathbf{k}, \tilde{\mathbf{p}})$  is an equilibrium, and let  $\tilde{\mathcal{A}}$  denote the set of equilibrium allocations of  $\tilde{\mathcal{E}}$ . Then  $\mathcal{A}_0 = \tilde{\mathcal{A}}$ . From  $\mathcal{A} = \mathcal{A}_0$ , it follows that the sets of equilibrium allocations are identical in  $\mathcal{E}$  and  $\tilde{\mathcal{E}}$ :  $\mathcal{A} = \tilde{\mathcal{A}}$ . This parallels the equilibrium equivalence result of DHKRS (Theorem 2, p. 618) for an economy with ORP (see also Dubey and Shubik (1985, p. 3)).

To make the model accessible to classical welfare analysis, assume that consumers' utility functions  $U_i(u_i, \theta_i, a_i, \mathbf{k}, \mathbf{v})$  are independent of the market valuations of the firms  $\mathbf{v}$ . Otherwise it is impossible to define Pareto optimality independently from market outcomes. Redefine an *allocation* in  $\mathcal{E}$  as  $((c_i, \theta_i, a_i, z_i)_{i=1}^I, \mathbf{k})$ , i.e., as a profile of consumption vectors and portfolios and a capital stock for each firm. An allocation is *feasible* if at date 0 and in all states at date 1, total consumption of each good does not exceed total production and total asset holdings equal total supply, i.e., if the goods market clearing conditions in Definition 5 hold with  $=$  replaced by  $\leq$  and the asset market clearing conditions hold.

**DEFINITION 7.** Let  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, \mathbf{k}^*)$  be a feasible allocation. Let  $((c_i)_{i=1}^I, \mathbf{k})$  satisfy the goods market clearing conditions in Definition 5 with  $=$  replaced by  $\leq$ . Suppose that if  $u_i(c_i, \mathbf{k}) \geq u_i(c_i^*, \mathbf{k}^*)$  for all  $i$  with strict inequality for some  $i$ , then there exists  $(\theta_i, a_i, z_i)_{i=1}^I$  such that  $((c_i, \theta_i, a_i, z_i)_{i=1}^I, \mathbf{k})$  is feasible and

$$U_i(u_i(c_i, \mathbf{k}), \theta_i, a_i, \mathbf{k}, \mathbf{v}) \geq U_i(u_i(c_i^*, \mathbf{k}^*), \theta_i^{**}, a_i^{**}, \mathbf{k}^*, \mathbf{v})$$

for all  $i$  with strict inequality for some  $i$ . Then we say that preferences satisfy *positive responsiveness*.

Positive responsiveness means that the overall utilities respond positively to consumption utilities: if an allocation Pareto-dominates in terms of consumption utilities (which take into account the externalities caused by firms), then the assets can be distributed across consumers in such a way that it also Pareto-dominates in terms of overall

<sup>7</sup>If the externalities caused by all producers  $j$  of good  $l$  enter all utility functions  $u_i$  only via total capital formation  $\sum_{j=1}^{J_l} k_{jl}$ , then  $t_{jl}$  is uniform across all producers of the good.

utilities (note the similarity to DHKRS's, p. 622, strong monotonicity property for ORP). Classification-based SRI implies positive responsiveness. This follows from the observation that the inequality in Definition 7 is satisfied for  $\theta_i = \theta_i^{**}$  and  $a_i = a_i^{**}$  if  $U_i$  is independent of  $k$ .

**THEOREM 3.** *Let  $U_i$  be independent of  $\mathbf{v}$  for all  $i$ , let the conditions of Theorem 2 hold, and let preferences satisfy positive responsiveness. Let  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, k^*)$  be a Pareto-optimal allocation in  $\mathcal{E}$ . Suppose there are  $t_{jl}$  ( $j = 1, \dots, J_l, l = 1, \dots, L$ ),  $T_i$  ( $i = 1, \dots, I$ ) and  $\tilde{\mathbf{p}}^*$  such that  $((c_i^*)_{i=1}^I, k^*, \tilde{\mathbf{p}}^*)$  is an equilibrium of  $\tilde{\mathcal{E}}$ . Then, given  $t_{jl}$  and  $T_i$ , there is an equilibrium  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, k^*, \mathbf{p}^*, \mathbf{v}^*, R^*, \mathbf{q}^*)$  of  $\mathcal{E}$ .<sup>8</sup>*

The theorem says that, given positive responsiveness, if implementation of a Pareto-optimal allocation is feasible with CCMs, then it is also feasible with financial markets and SRI, using the same Pigouvian taxes and lump-sum transfers. The price mechanism, including transfers and taxes, allocates productive capital, consumption goods, and financial assets efficiently.

The proof of Theorem 3 makes use (following DHKRS's, p. 622ff, proof of the Second Welfare Theorem with ORP) of the fact that the set of Pareto-optimal allocations of  $\mathcal{E}$  is a subset of the set of Pareto-optimal allocations of  $\mathcal{E}_0$ . This is not generally true if positive responsiveness is violated. In that case asset holdings can be valued so strongly in  $\mathcal{E}$  that an allocation with low consumption utilities is Pareto-optimal, even though it is not Pareto-optimal in  $\mathcal{E}_0$ .

### 4.3 Real effects of SRI

According to Theorems 1 and 2, separability, SV maximization, and SWANS are jointly sufficient for neutrality of SRI. This subsection discusses the implications of giving up the former two conditions. Market incompleteness is analyzed in Section 5.

In principle, consumers can affect corporate behavior in two ways: by changing the cost of capital or the demands for goods. The separability assumption allows it to disentangle these two channels. Theorems 1 and 2 show that if the goods demand channel is shut down, then, given market completeness, there is no effect via the cost-of-capital channel either. That is, so as to have positive impact, pro-social consumer behavior must not be confined to portfolio formation, but must also have an impact on the relative demands for goods.

Given separability and market completeness, SRI does not provide incentives for SV maximizing firms to change their behavior. However, initial shareholders are of course free to incorporate other factors besides SV in firms' objective functions, which give rise to different corporate behavior. That is, even if corporate social responsibility (CSR) cannot be enforced via the stock exchange, it can be implemented by incumbent shareholders of their own accord (for a survey of CSR, see Kitzmüller and Shimshack (2012)).

<sup>8</sup>The conditions of the theorem, with positive responsiveness replaced by the stronger assumption of classification-based SRI, also imply the validity of the First Welfare Theorem in the absence of externalities.

An objection to this view is “amoral drift” (Hart and Zingales (2017, p. 255)): in theory, if a firm does not maximize SV and the firm’s capital stock is not fixed irreversibly, a hostile takeover is an arbitrage opportunity, so there are strong financial incentives for neutral investors to re-implement SV maximization. To see this, consider an equilibrium at which the initial owners of a firm  $(j, l)$  with a different objective than SV maximization decide to install a capital stock  $k_{jl}$  that falls short of the level  $k'_{jl}$  that maximizes  $v_{jl} - (p_0 k_{jl} - R b_{jl})$ . Consider the following takeover strategy targeted at the firm: buy all shares of the firm at their market value  $v_{jl}$ , install additional capital  $k'_{jl} - k_{jl}$  at cost  $p_0(k'_{jl} - k_{jl})$ , and sell a portfolio that pays off  $p_{ls} f_{jls}(k'_{jl}) - b_{jl}$  short (the existence of such a portfolio is guaranteed by market completeness). The cash flow generated by the firm at date 1 is sufficient to unwind the short positions in each state  $s$ . The date-0 revenue from the short sales  $\sum_{s=1}^S r_s [p_{ls} f_{jls}(k'_{jl}) - b_{jl}] (= v'_{jl})$  coincides with the market capitalization of the firm under SV maximization. The net payoff at date 0 is positive exactly if

$$v'_{jl} > v_{jl} + p_0(k'_{jl} - k_{jl}).$$

The validity of this inequality follows from the fact that  $k'_{jl}$  maximizes  $v_{jl} - p_0(k_{jl} - R b_{jl})$ , whereas  $k_{jl}$  does not.

## 5. INCOMPLETE MARKETS

This section analyzes the incomplete markets case in which SWANS is violated. SRI is neutral if for each firm a consumer cares about, there is another firm with perfectly correlated productivity that she does not care about, i.e., if the payoffs of portfolios including stocks targeted by SRI can be replicated for given holdings of these stocks despite incompleteness. A CAPM special case of the model is used to show that the impact of SRI on real economic activity is small if there are closely correlated substitute stocks.

### 5.1 Model

We adopt the simplifying assumptions of Diamond’s (1967) classical GEI model. There is only one good at date 1 (i.e.,  $L = 1$ ). We drop the index for goods  $l$  and normalize the spot prices to unity ( $p_0 = 1$  and  $p_s = 1$ ,  $s = 1, \dots, S$ ). The firms’ production functions feature *multiplicative uncertainty*:  $y_{js} = \lambda_{js} f_j(k_j)$ , where  $\lambda_j$  is a productivity shock with positive realization  $\lambda_{js}$  in each state  $s$ .<sup>9</sup> They are twice continuously differentiable with  $f'_j(k_j) > 0 > f''_j(k_j)$  for all  $k_j > 0$ ,  $f'_j(k_j) \rightarrow \infty$  for  $k_j \rightarrow 0$ , and  $f'_j(k_j) \rightarrow 0$  for  $k_j \rightarrow \infty$ . Firms do not issue bonds, so  $a_i$  drops out of the utility functions  $U_i$ , which is assumed differentiable. Consumer  $i$ ’s consumption utility is

$$u_i(c_i, \mathbf{k}) = \{v_i(c_{i0}) + \beta_i E[v_i(c_i)]\} e_i(\mathbf{k}),$$

where  $E$  is the expectations operator,  $c_i$  is  $i$ ’s random date-1 consumption, and  $\beta_i (> 0)$  is her subjective discount factor. The function  $v_i$  is twice continuously differentiable with

<sup>9</sup>Here and in what follows,  $\xi$  denotes the random variable with realizations  $\xi_s$  ( $s = 1, \dots, S$ ).



$v'_i(c) > 0 > v''_i(c)$  for all  $c > 0$  and  $v'_i(c) \rightarrow \infty$  for  $c \rightarrow 0$ . The function  $e_i(\mathbf{k})$  is positive-valued and captures the (non-state-dependent) effect of externalities on  $i$ 's consumption utility. The only available security is a safe asset with payoff  $x_{1s} = 1$ . Its price is denoted ( $q_1 =$ )  $q$  and  $i$ 's holdings are denoted as ( $z_{i1} =$ )  $z_i$ . We assume that  $\lambda_j$  ( $j = 1, \dots, J$ ) and the safe asset do not span the state space, so that SWANS is violated and the market is incomplete (a simple sufficient condition is  $J + 1 < S$ ). The economy is denoted as  $\mathcal{X}$  and its special case without SRI as  $\mathcal{X}_0$ .

### 5.2 Equilibrium

The necessary conditions for a utility maximizing choice of  $\theta_i$  and  $z_i$  are

$$\frac{\partial U_i}{\partial u_i} \{-v'_i(c_{i0})v_j + E[\beta_i v'_i(c_i)y_j]\} e_i(\mathbf{k}) + \frac{\partial U_i}{\partial \theta_{ij}} = 0, \quad j = 1, \dots, J \tag{7}$$

$$\frac{\partial U_i}{\partial u_i} \{-v'_i(c_{i0})q + E[\beta_i v'_i(c_i)]\} e_i(\mathbf{k}) = 0. \tag{8}$$

From (7) and (8), the asset prices satisfy

$$v_j = f_j(k_j)E(\tilde{m}_{ij}\lambda_j) \tag{9}$$

and  $q = E(m_i)$ , where

$$\tilde{m}_{ij} = \frac{m_i}{1 - \frac{\partial U_i / \partial \theta_{ij}}{v'_i(c_{i0})e_i v_j \partial U_i / \partial u_i}} \tag{10}$$

and  $m_i = \beta_i v'_i(c_i) / v'_i(c_{i0})$ . Consumer  $i$  applies different stochastic discount factors (SDFs)  $\tilde{m}_{ij}$  to the payoffs of different firms  $j$ . Her SDF for  $j$  is higher or lower than the standard SDF  $m_i$ , depending on whether  $\partial U_i / \partial \theta_{ij}$  is positive or negative, i.e., whether  $i$  prefers an increase or a decrease in her share in  $j$ , respectively. When  $U_i$  satisfies satiation in SRI and  $\theta_{ij}$  maximizes utility, so that  $\partial U_i / \partial \theta_{ij} = 0$ , the SDF boils down to  $\tilde{m}_{ij} = m_i$ .

As in the complete markets model, we assume SV maximization under *competitive expectations*. That is, firm  $j$  maximizes  $v_j - k_j$  with  $v_j$  given by (9) taking the equilibrium value of  $\tilde{m}_{ij}$  as given. The optimum capital stock then satisfies

$$f'_j(k_j)E(\tilde{m}_{ij}\lambda_j) = 1. \tag{11}$$

From (9),  $E(\tilde{m}_{ij}\lambda_j)$  ( $= v_j / f_j(k_j)$ ) is uniform across  $i$  in equilibrium, so the outcome of SV maximization is independent of whose consumer's SDF  $j$  uses to value its cash flow  $y_j$ . SV maximization can be justified similarly as in Section 3.3: each firm  $j$  is controlled by initial shareholders  $i$  whose utilities  $U_i$  are independent of  $\theta_i$ ,  $\mathbf{k}$ , and  $\mathbf{v}$ , or  $U_i$  is given by (6) with  $D_{ij} = 1$  for all  $i$ . This follows from the next lemma.

LEMMA 5. For  $U_i$  given by (6) with  $D_{ij} = 1$  ( $i = 1, \dots, I$ ,  $j = 1, \dots, J$ ), holding  $e_i(\mathbf{k})$  and  $v_M$  constant,

$$\frac{dU_i}{dk_j} = \frac{\partial U_i}{\partial u_i} v'_i(c_{i0}) e_i \bar{\theta}_{ij} \left( \frac{dv_j}{dk_j} - 1 \right) + \left( -\frac{\theta_{ij}}{v_j} \frac{\partial U_i}{\partial \theta_{ij}} + \frac{\partial U_i}{\partial v_j} \right) \frac{dv_j}{dk_j} + \frac{\partial U_i}{\partial k_j}. \tag{12}$$

For indifferent initial shareholders  $i$ , the latter three partial derivatives in (12) vanish, so SV maximization (i.e.,  $dv_j/dk_j = 1$ ) maximizes utility. This is the basis of Diamond's (1967) classical shareholder unanimity theorem (see also Magill and Quinzii (2002), p. 418). With overall utility given by (6) and  $D_{ij} = 1$ ,

$$\frac{\theta_{ij}}{v_j} \frac{\partial U_i}{\partial \theta_{ij}} = \theta_{ij} \frac{\partial \hat{U}_i}{\partial (\theta_{ij} v_{ij})} = \frac{\partial U_i}{\partial v_j}$$

and  $\partial U_i / \partial k_j = 0$ . Again,  $dv_j/dk_j = 1$  implies  $dU_i/dk_j = 0$ .

**DEFINITION 8.** The vector  $((c_i, \theta_i, z_i)_{i=1}^I, \mathbf{k}, \mathbf{v}, q)$  is an *equilibrium* of  $\mathcal{X}$  if  $(c_i, \theta_i, z_i)$  maximizes  $U_i$  subject to (1) and (2),  $k_j$  maximizes  $v_j - k_j$  with  $v_j$  given by (9) taking  $\hat{m}_{ij}$  for some  $i$  as given, and the goods and asset markets clear.

### 5.3 Neutrality of SRI

It is easy to see that the neutrality result from the complete markets case does not generally hold if SWANS is violated. Consider the following example. All consumers have the same endowments  $y_{i0}$ , the same initial shareholdings  $\hat{\theta}_{ij} (= 1/I)$ , and the same consumption utility functions  $u_i$ . There is a no-trade equilibrium of  $\mathcal{X}_0$  in which consumption  $c_{is}$  is uniform across consumers  $i$  in each state  $s$ . Suppose there is a pair of states  $(s, s')$  such that the set of all firms can be partitioned into two subsets  $\mathcal{J}'$  and  $\mathcal{J}''$  with  $\lambda_{js} = \lambda_{j's'}$  for all  $j \in \mathcal{J}'$  and  $\lambda_{js} > \lambda_{j's'}$  for all  $j \in \mathcal{J}''$ . That is, the firms in  $\mathcal{J}'$  are equally productive in states  $s$  and  $s'$ , whereas the firms in  $\mathcal{J}''$  are more productive in  $s$  than in  $s'$ . Then  $c_{is}/c_{is'} > 1$  for all  $i$  at the no-trade equilibrium of  $\mathcal{X}_0$ . In  $\mathcal{X}$ , let SR consumer  $i$  have a strong aversion to nonzero holdings of the firms  $j \in \mathcal{J}''$ , so that  $\theta_{ij} = 0$ . Since all other assets have the same payoff in  $s$  as in  $s'$ ,  $c_{is}/c_{is'} = 1$  at an equilibrium of  $\mathcal{X}$ . SRI is nonneutral. As  $i$  refuses to hold shares of firms in  $\mathcal{J}''$ , she cannot benefit from the high returns on the stocks of these firms in state  $s$  and, therefore, she cannot maintain her  $\mathcal{X}_0$  consumption profile in  $\mathcal{X}$ .

SRI is neutral despite incompleteness if, contrary to the example, it does not affect SR consumers' consumption possibilities. The following result provides a simple example:

**THEOREM 4.** Let all  $U_i$  satisfy satiation in SRI, and let the set of neutral consumers be non-empty. Suppose for each firm  $j$  consumer  $i$  cares about, there is a firm  $j'$  she is indifferent toward with  $\lambda_{js} = \lambda_{j's}$  for all  $s$ . Then, if  $((c_i^*, \theta_i^*, z_i^*)_{i=1}^I, \mathbf{k}^*, \mathbf{v}^*, q^*)$  is an equilibrium of  $\mathcal{X}$ , it is an equilibrium of  $\mathcal{X}_0$ . Conversely, if  $((c_i^*, \theta_i^{**}, z_i^{**})_{i=1}^I, \mathbf{k}^*, \mathbf{v}^*, q^*)$  is an equilibrium of  $\mathcal{X}_0$ , there is an equilibrium  $((c_i^*, \theta_i^*, z_i^*)_{i=1}^I, \mathbf{k}^*, \mathbf{v}^*, q^*)$  of  $\mathcal{X}$ .

Each consumer  $i$  can replicate the payoff on any portfolio of all stocks with pairwise perfectly correlated outputs with an alternative portfolio of the same stocks that includes given quantities of the stocks she cares about at no change in cost (see Lemma 6 in the Appendix). So  $i$  chooses the same consumption vector  $c_i^*$  as in  $\mathcal{X}_0$  and a portfolio

with  $\partial U_i / \partial \theta_{ij} = 0$  for all  $j$  she cares about in  $\mathcal{X}$ . As a result, the SDFs and the SV maximizing capital stocks are the same in  $\mathcal{X}$  and  $\mathcal{X}_0$ . The only difference in the equilibria is that within each set, all stocks with pairwise perfectly correlated outputs, SR consumers' portfolios are determinately tilted toward pro-social stocks in  $\mathcal{X}$ .

If the equilibrium  $((c_i^*, \theta_i^*, z_i^*)_{i=1}^I, k^*, v^*, q^*)$  of  $\mathcal{X}$  is a continuous function of (a parameter that controls) the correlation between stocks SR consumers care about and their closest substitutes, then the real effects of SRI are small when there are close substitutes for stocks SR consumers care about. A CAPM example is presented in the subsequent subsection.

Financial market completeness is thus not a necessary condition for neutrality or almost neutrality of SRI. What matters is that SR consumers can replicate or nearly replicate the payoffs of stocks they care about for social responsibility reasons using other assets at no additional cost.

### 5.4 CAPM

This subsection uses the CAPM special case of the incomplete markets model to illustrate almost neutrality for closely correlated returns and to show how SRI affects the cross section of returns and real economic activity under alternative conditions (cf. the CAPMs with exogenous outputs in Luo and Balvers (2017), Zerbib (2020), Pedersen, Fitzgibbons, and Pomorski (2021), and Pastor, Stambaugh, and Taylor (2021)).

For each consumer  $i$ , let  $v_i(c) = -\exp(-\rho_i c)$ , where  $\rho_i (>0)$  is her coefficient of absolute risk aversion. Consumer  $i$ 's overall utility function is

$$U_i(u_i(c_i, k), \theta_i, k, v) = u_i(c_i, k) \exp \left[ - \sum_{j=1}^J \gamma_{ij}(\theta_{ij} v_j) \right],$$

where the  $\gamma_{ij}$ s are differentiable functions. As  $u_i(c_i, k) < 0$ , the higher is  $\gamma_{ij}(\theta_{ij} v_j)$ , the higher is  $i$ 's overall utility  $U_i$ . A negative value of  $\gamma_{ij}(\theta_{ij} v_j)$  thus indicates that  $i$  dislikes firm  $j$  and vice versa. This utility function is a special case of (6) with  $D_{ij} = 1$ , so, from Lemma 5, there is unanimity about SV maximization. Using  $E(m_i) = q$ ,  $i$ 's SDF for firm  $j$  in (10) becomes

$$\tilde{m}_{ij} = \frac{m_i}{1 - \frac{1+q}{\rho_i} \gamma'_{ij}(\theta_{ij} v_j)}. \tag{13}$$

It is high if an increase in her holdings of stocks of  $j$  raises  $\gamma_{ij}(\theta_{ij} v_j)$  and  $U_i$ , and vice versa.

Suppose the productivity shocks  $\lambda_j$  are jointly normally distributed (assuming now that the state space is given by the real line and negative output and consumption levels are admissible). Using  $E(m_i) = q$  and Stein's lemma for normal random variables (i.e.,  $\text{cov}[y_j, \exp(-\rho c_i)] = \sum_{j'=1}^J \text{cov}(y_j, y_{j'}) E\{\partial[\exp(-\rho c_i)] / \partial y_{j'}\}$ ), (13) can be rewritten as

$$\sum_{j'=1}^J \text{cov}(y_j, y_{j'}) \theta_{ij'} = \frac{1}{\rho_i} \left[ E(y_j) - \frac{v_j}{q} \right] + \frac{1+q}{q} \frac{\gamma'_{ij}(\theta_{ij} v_j)}{\rho_i^2} v_j. \tag{14}$$

Let

$$\gamma_{ij}(\theta_{ij}v_j) = g_{ij} \frac{(\theta_{ij}v_j)^2}{2},$$

where  $g_{ij} = g < 0$  for  $i$  and  $j$  odd, and  $g_{ij} = 0$  otherwise. That is, the odd-numbered consumers are socially responsible and dislike holding nonzero amounts of shares of odd-numbered firms. There is satiation in SRI. The productivity shocks are  $\lambda_j = \lambda$  for  $j$  odd and  $\lambda_j = \lambda + \kappa\varepsilon$  for  $j$  even, where  $\lambda$  and  $\varepsilon$  are independent normal random variables with positive variances and  $\kappa \geq 0$ . As  $\kappa \rightarrow 0$ , the correlation between the productivity shocks of odd- and even-numbered firms approaches 1. In the limit, the conditions of Theorem 4 are satisfied. All consumers are characterized by the same endowment  $y_{i0} = y_0$ , the same subjective discount factor  $\beta_i = \beta$ , and the same degree of absolute risk aversion  $\rho_i = \rho$ ;  $f_j = f$  is uniform across all  $j$ ;  $I$  and  $J$  are even.

Eliminating  $\theta_{12} - \theta_{22}$  from (14) for  $j = 1$  and  $j = 2$ , and using  $(I/2)(\theta_{11} + \theta_{21}) = 1$  yields

$$J[f(k_1)]^2 \kappa^2 \frac{\sigma_\lambda^2 \sigma_\varepsilon^2}{\sigma_\lambda^2 + \kappa^2 \sigma_\varepsilon^2} \left( \theta_{11} - \frac{1}{I} \right) = \frac{1+q}{q} \frac{g\theta_{11}}{\rho^2} v_1^2.$$

For  $\kappa$  close to 0,  $\theta_{11} \approx 0$  and  $\theta_{21} \approx 2/I$ . By symmetry,  $\theta_{1j} = \theta_{11}$  and  $\theta_{2j} = \theta_{21}$  for all  $j$  odd. That is, as the stocks become perfect substitutes, the stocks SR consumers dislike are exclusively held by the neutral consumers. As  $\gamma'_{ij}(\theta_{ij}v_j) \approx 0$ , the SDFs  $\tilde{m}_{ij} \approx m_i$  are approximately uniform across firms, so SV maximization implies  $k_1 \approx k_2$ . So  $\theta_{12} - \theta_{22} \approx -(\theta_{11} - \theta_{21})$ . From market clearing and symmetry,  $\theta_{1j} = \theta_{12} \approx 2/I$  and  $\theta_{2j} = \theta_{22} \approx 0$  for all  $j$  even. That is, the SR consumers hold all the stocks they are indifferent toward. Substituting  $c_{i0} \approx y_0 - (J/I)k_j$  and  $c_i \approx (J/I)\lambda f(k_j)$  into (13) with  $\gamma'_{ij}(\theta_{ij}v_j) \approx 0$  and the resulting SDF  $\tilde{m}_{ij}$  into (11) shows that the uniform equilibrium capital stock  $k_j$  is approximately equal to the value determined by

$$f'(k_j) \beta E \left( \exp \left\{ -\rho \left[ \frac{J}{I} \lambda f(k_j) - y_0 + \frac{J}{I} k_j \right] \right\} \lambda \right) = 1,$$

which is also the equilibrium capital stock in  $\mathcal{X}_0$ . Thus, SRI is almost neutral when the correlation between SRI stocks and non-SRI stocks is close to unity.

Alternatively, consider the following specification with non-satiation in SRI, borrowed from Pastor, Stambaugh, and Taylor (2021):

$$\gamma_{ij}(\theta_{ij}v_j) = g_{ij} \theta_{ij} v_j.$$

Suppose there is no pair of firms,  $j$  and  $j'$ , with identical productivity shocks (i.e., with  $\lambda_{js} = \lambda_{j's}$  for all  $s$ ). Otherwise an equilibrium does not exist (see the proof of Theorem 4 in the Appendix). Using market clearing (i.e.,  $\sum_{i=1}^I \theta_{ij} = 1$ ), it follows from (14) that

$$\sum_{j'=1}^J \text{cov}(y_j, y_{j'}) = \sum_{i=1}^I \frac{1}{\rho_i} \left[ E(y_j) - \frac{v_j}{q} \right] + \frac{1+q}{q} \Gamma_j v_j,$$

where

$$\Gamma_j = \sum_{i=1}^I \frac{g_{ij}}{\rho_i^2}$$

is a weighted sum of all consumers' attitudes toward firm  $j$  ( $= 1, \dots, J$ ). Define  $r_j = y_j/v_j - 1$  as the return on stock  $j$ ,  $r_F = 1/q - 1$  as the safe interest rate,  $r_M = (\sum_{j=1}^J y_j)/v_M - 1$  as the return on the market, and  $\sigma_{jM} = \text{cov}(r_j, r_M)$  as the covariance between  $j$ 's return and the market return. Then

$$\frac{E(r_j) - r_F}{E(r_M) - r_F} = \frac{\sigma_{jM} - \frac{2 + r_F}{v_M} \Gamma_j}{\sigma_M^2 - \frac{2 + r_F}{v_M} \sum_{j'=1}^J \frac{v_{j'}}{v_M} \Gamma_{j'}}$$

The special case with  $g_{ij} = 0$  for all  $i$  and  $j$  yields the standard CAPM formula. SRI turns the model into a two-factor model. Firms  $j$  with a negative assessment by the market ( $\Gamma_j$  negative) must pay investors a return premium and vice versa. This follows straightforwardly from the logic of SDFs. Suppose, starting from an equilibrium of  $\mathcal{X}_0$ , a subset of consumers take a critical stance on firm  $j$  (so that  $\gamma'_{ij}(\theta_{ij}v_j) = g_{ij} < 0$ ). From (13), this implies additional discounting of the payoffs of stocks  $j$  with a negative ESG assessment (a lower value of  $E(\tilde{m}_{ij})$ ). The SR consumers sell some of these stocks to indifferent consumers. This reduces the covariance of  $c_i$  and  $\lambda_j$  for the sellers and increases it for the buyers, thereby restoring equality of  $E(\tilde{m}_{ij}\lambda_j)$  across all consumers (see (9)). At the resulting equilibrium of  $\mathcal{X}$ ,  $v_j$  is lower than in  $\mathcal{X}_0$ , because both SR consumers (due to  $g_{ij} < 0$ ) and indifferent consumers (due to purchases of stocks of  $j$ ) discount  $j$ 's cash flow more strongly.

From (9), (11), and the definition of the return on  $j$ ,

$$E(r_j) = E(\lambda_j)f'_j(k_j) - 1.$$

Given  $E(\lambda_j)$ ,  $k_j$  is inversely related to  $E(r_j)$  and, hence, positively related to  $\Gamma_j$ . That is, the negative cross-sectional relation between firms' returns and ESG performance also identified in SRI-augmented CAPMs with exogenous outputs leads to a positive cross-sectional relation between capital formation and ESG performance.

## 6. EMPIRICAL IMPLICATIONS

The model has several empirical implications for the relation between SRI, asset returns, and real economic activity. Generally, it is consistent with mixed existing results about the neutrality or nonneutrality of SRI. More specifically, it yields several testable hypotheses about factors that have a bearing on whether SRI is related to real economic activity or not.

Given market completeness and SV maximization, the efficacy of SRI depends on nonseparability of SR consumers' utility functions, i.e., on a positive relation between sustainable investing and sustainable consumption (see Section 4.3). As rising demands

for pro-social goods tend to drive up their prices, SR consumers' additional demand crowds out neutral consumers' demand. Theory thus predicts that SRI is related to financial returns and real economic activity if interacted with a measure of the correlation between socially responsible investments and socially responsible consumption at the household level.

Given separability and SV maximization, SRI is neutral if SWANS is satisfied (see Section 4.1). This is a weak condition in theory. It is satisfied when each consumer can form a portfolio with different payoffs in each state using assets she is indifferent toward and it is possible to write simple call options with arbitrary strike prices on that portfolio (see Ross (1976)). The instruments needed for spanning become more complicated in dynamic settings (see Bajeux-Besnainou and Rochet (1996)) or when the state space is multidimensional (see Pazarbasi, Schneider, and Vilkov (2021)), however. In addition, for practical purposes, it is a strong assumption that insurance against any conceivable contingency using options or other securities is feasible and used comprehensively to neutralize the impact of SRI on consumption profiles. So theory predicts that SRI is related to financial returns and capital formation if interacted with the sophistication of financial markets and the intensity with which SR consumers use them. Pazarbasi, Schneider, and Vilkov (2021, p. 28) propose several such measures, derived from bid–ask spreads, moneyness ranges, volumes of outstanding options, and numbers of option trades, in a different context.<sup>10</sup> An illustrative piece of anecdotal evidence in favor of a link between market incompleteness and the efficacy of SRI is Renneboog, Ter Horst, and Zhang's (2008, p. 308) finding that the differences in the alphas of SRI funds versus conventional funds are negative and significant in several continental European and Asian countries, but not in the United States, where financial markets are arguably most developed.

Given market completeness and separability, pro-social corporate policies have to be initiated by incumbent shareholders, rather than outside investors, to succeed. However, if successful, a hostile takeover is an arbitrage opportunity, which raises the problem of amoral drift back to nonresponsible corporate policies (see Section 4.3). So the model predicts that pro-social corporate reforms are more frequent in markets with strong takeover protection, such as anti-takeover legislation or poison pills, for instance (see Hart and Zingales (2017, p. 256)). Emmanuel Faber's resignation as chief executive officer (CEO) of Danone is a recent example. Danone underperformed financially after Faber put CSR goals high on its agenda. This provided incentives for ("wolf packs" of) hedge funds to acquire stakes below disclosure thresholds up to a point where they could credibly threaten to dismiss the CEO and enforce a firm policy that puts less weight on CSR goals.<sup>11</sup>

Even if financial markets are incomplete, the impact of SRI on real economic activity is small if consumers can neutralize the impact of SRI on their portfolio payoffs using closely correlated stocks or portfolios of stocks (see Section 5.4). Theory thus predicts

<sup>10</sup>Pazarbasi, Schneider, and Vilkov (2021) investigate the recovery of bounds on the dispersion of beliefs from observed asset prices.

<sup>11</sup>See <https://www.forbes.com/sites/timabansal/2021/04/15/how-hedge-funds-are-destroying-corporations-and-society/?sh=7420b7077ead>.

that SRI is related to financial returns and real economic activity if interacted with a measure of how well stocks' returns can be replicated using close substitutes. Leaving the confines of the model with multiplicative shocks, consider a set of ex ante similar, competing firms in an energy-intensive industry. Suppose reforms that enhance social performance or promote good governance reduce the firms' payoffs proportionately in all states, whereas, due to differential exposure to energy price shocks, environment-friendly reform changes the payoff pattern in such a way that stock returns are harder to replicate using competitors' shares. Theory predicts that SRI is more strongly related to real economic activity if it aims at "E" reforms, rather than "S or G" reforms. This is in contrast, however, to Pedersen, Fitzgibbons, and Pomorski's (2021, p. 592) findings that there are positive abnormal returns for stocks with low accruals (a proxy for unaggressive accounting and good governance), but not for low-carbon intensity stocks.

The CAPM special case of the incomplete markets model in Section 5.4 yields a two-factor pricing equation that is similar to the SRI-augmented CAPMs of Luo and Balvers (2017), Zerbib (2020), Pedersen, Fitzgibbons, and Pomorski (2021), and Pastor, Staambaugh, and Taylor (2021), and, therefore, serves as an alternative motivation for existing regressions of returns on measures of SRI. Our model predicts that returns are inversely related to capital formation (see Section 5.4). It would be interesting to see whether the firm data underlying return regressions conform to this pattern.

## 7. CONCLUSION

Even having reached multi-trillion dollar volume, it cannot be taken for granted that SRI has real effects. We provide a set of jointly sufficient conditions for neutrality: unrestricted spanning opportunities provided by assets that cause no SRI concerns, separability of utility, and shareholder value maximization. SRI is neutral despite incompleteness of the financial market if there are perfect substitutes for targeted stocks. This is the main message of the paper: financial activities aimed at fostering sustainability, well meant, implemented by professional funds, and promoted by organizations and politics, are not necessarily effective.

The framework of our analysis is the cornerstone equilibrium model of resource allocation and consumption-based asset pricing. This setup allows us to precisely identify the circumstances under which SRI has real effects on returns and capital formation. Additionally, it can be used to address further questions that arise in the analysis, for instance, "how do the decision making processes in firms impact the efficacy of SRI when the conditions for shareholder unanimity are violated" and "does SRI lead to financial innovations that enable consumers to neutralize the impact of SRI on consumption when markets are incomplete?"

Our results provide an explanation for mixed results of empirical studies of SRI. In addition, they yield the testable empirical hypothesis that SRI is related to returns and to real economic activity if interacted with variables that proxy for the theoretical factors that have a bearing on the efficacy of SRI, viz., the correlation between SRI and sustainable consumption at the household level, financial market completeness, and the correlation between targeted stocks and substitutes.

APPENDIX

Let  $\mathcal{J} = \{(j, l) | j = 1, \dots, J_l, l = 1, \dots, L\}$ ,  $\mathcal{J}_i = \{(j, l) | j = 1, \dots, J_l, l = 1, \dots, L, i \text{ is indifferent toward } (j, l)\}$ , and  $\mathcal{J}_i^c = \mathcal{J} \setminus \mathcal{J}_i$ .

**PROOF OF LEMMA 1.** If there is a neutral consumer  $i$ , then, given SWANS, a unique vector of state prices  $\mathbf{r}$  exists that satisfies (3)–(5).

The same  $\mathbf{r}$  is also the unique state price vector in the absence of a neutral consumer. To see this, let  $i$  hold arbitrary given amounts of assets  $(j, l) \in \mathcal{J}_i^c$ . Because of SWANS, the assets  $(j, l) \in \mathcal{J}_i$  span the state space. So the absence of arbitrage opportunities using assets  $(j, l) \in \mathcal{J}_i$  for  $i$  implies that there exists a unique vector of state prices  $\mathbf{r}_i$  that satisfies (3) for  $(j, l) \in \mathcal{J}_i$ , (4), and (5). The fact that  $\mathbf{r}$  satisfies these equations and uniqueness of  $\mathbf{r}_i$  imply  $\mathbf{r}_i = \mathbf{r}$ . That  $\mathbf{r}$  is the unique solution to (3) for all  $(j, l) \in \mathcal{J}$ , (4), and (5) follows from the assumption that for each firm  $(j, l)$ , there is  $i$  such that  $(j, l) \in \mathcal{J}_i$  (so that  $\cup_{i=1}^I \mathcal{J}_i = \mathcal{J}$ ).  $\square$

**PROOF OF LEMMA 2.** Suppose  $(\mathbf{c}_i, \boldsymbol{\theta}_i, \mathbf{a}_i, \mathbf{z}_i)$  satisfies (1) and (2) with equality. This is without loss of generality, because otherwise there is  $\mathbf{c}'_i \geq \mathbf{c}_i$  that satisfies (1) and (2) with equality and the lemma establishes affordability of  $\mathbf{c}'_i$ , which implies affordability of  $\mathbf{c}_i$ . From SWANS, there is  $(\boldsymbol{\theta}'_i, \mathbf{a}'_i, \mathbf{z}'_i)$  with  $\theta'_{ijl}$  and  $a'_{ijl}$  given for  $(j, l) \in \mathcal{J}_i^c$  that yields payoff  $\sum_{l=1}^L p_{ls} c_{ils}$  in  $s$ , i.e.,

$$\sum_{l=1}^L p_{ls} c_{ils} - \sum_{l=1}^L \sum_{j=1}^{J_l} [\theta'_{ijl} (p_{ls} y_{jls} - b_{jl}) + a'_{ijl}] - \sum_{m=1}^M x_{ms} z'_{im} = 0.$$

Let  $(\Delta \boldsymbol{\theta}_i, \Delta \mathbf{a}_i, \Delta \mathbf{z}_i) = (\boldsymbol{\theta}'_i, \mathbf{a}'_i, \mathbf{z}'_i) - (\boldsymbol{\theta}_i, \mathbf{a}_i, \mathbf{z}_i)$ . Using (2) with equality,

$$\sum_{l=1}^L \sum_{j=1}^{J_l} [\Delta \theta_{ijl} (p_{ls} y_{jls} - b_{jl}) + \Delta a_{ijl}] + \sum_{m=1}^M x_{ms} \Delta z_{im} = 0.$$

Multiply by  $r_s$ , add up, and use (3)–(5) to get

$$\sum_{l=1}^L \sum_{j=1}^{J_l} (v_{jl} \Delta \theta_{ijl} + R \Delta a_{ijl}) + \sum_{m=1}^M q_m \Delta z_{im} = 0. \tag{A.1}$$

Since  $(\mathbf{c}_i, \boldsymbol{\theta}_i, \mathbf{a}_i, \mathbf{z}_i)$  satisfies (1) with equality,

$$\begin{aligned} p_0 (c_{i0} - y_{i0}) + \sum_{l=1}^L \sum_{j=1}^{J_l} [\bar{\theta}_{ijl} (p_0 k_{jl} - R b_{jl}) + (\theta'_{ijl} - \Delta \theta_{ijl} - \bar{\theta}_{ijl}) v_{jl} \\ + R (a'_{ijl} - \Delta a_{ijl})] + \sum_{m=1}^M q_m (z'_{im} - \Delta z_{im}) = 0. \end{aligned}$$

Jointly with (A.1), it follows that  $(\mathbf{c}_i, \boldsymbol{\theta}'_i, \mathbf{a}'_i, \mathbf{z}'_i)$  satisfies (1) with equality.  $\square$



PROOF OF LEMMA 3. Let  $(c_i^*, \theta_i^*, a_i^*, z_i^*)$  maximize  $U_i(u_i(c_i, k), \theta_i, a_i, k, v)$  subject to (1) and (2). Suppose there is  $(c_i, \theta_i, a_i, z_i)$  satisfying (1) and (2) such that  $u_i(c_i, k) > u_i(c_i^*, k)$ . From Lemma 2,  $c_i$  can be achieved with  $\theta_{ijl}^*$  and  $a_{ijl}^*$  for  $(j, l) \in \mathcal{J}_i^c$ . The ensuing overall utility is  $U_i(u_i(c_i, k), \theta_i^*, a_i^*, k, v)$ . From strong monotonicity,

$$U_i(u_i(c_i, k), \theta_i^*, a_i^*, k, v) > U_i(u_i(c_i^*, k), \theta_i^*, a_i^*, k, v).$$

This contradicts the fact that  $(c_i^*, \theta_i^*, a_i^*, z_i^*)$  maximizes  $U_i(u_i(c_i, k), \theta_i, a_i, k, v)$ .

Let  $(c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})$  maximize  $u_i(c_i, k)$  subject to (1) and (2). From Lemma 2,  $i$  can achieve  $c_i^*$  with given  $\theta_{ijl}$  and  $a_{ijl}$  for  $(j, l) \in \mathcal{J}_i^c$ . Suppose she chooses  $(\theta_i^*, a_i^*, z_i^*)$  such that  $\theta_{ijl}^*$  and  $a_{ijl}^*$  for  $(j, l) \in \mathcal{J}_i^c$  maximize  $U_i(u_i(c_i^*, k), \theta_i, a_i, k, v)$ . Satiation in SRI ensures the existence of a maximum. Then

$$U_i(u_i(c_i^*, k), \theta_i^*, a_i^*, k, v) \geq U_i(u_i(c_i^*, k), \theta_i, a_i, k, v)$$

for all  $\theta_i$  and  $a_i$  that jointly with  $c_i^*$  and some  $z_i$  satisfy (1) and (2). The lemma asserts that there is no  $(c_i, \theta_i, a_i, z_i)$  satisfying (1) and (2) that yields higher utility than  $(c_i^*, \theta_i^*, a_i^*, z_i^*)$ . Suppose to the contrary that

$$U_i(u_i(c_i, k), \theta_i, a_i, k, v) > U_i(u_i(c_i^*, k), \theta_i^*, a_i^*, k, v)$$

for some  $(c_i, \theta_i, a_i, z_i)$  that satisfies (1) and (2). Together with the preceding inequality, it follows from monotonicity that  $u_i(c_i, k) > u_i(c_i^*, k)$ , a contradiction.  $\square$

PROOF OF LEMMA 4. Let  $k_{jl}$  maximize  $v_{jl} - (p_0 k_{jl} - R b_{jl})$  and let  $(c_i, \theta_i, a_i, z_i)$  maximize  $\hat{U}_i$  subject to (1) and (2). From Lemma 3,  $(c_i, \theta_i, a_i, z_i)$  maximizes  $u_i(c_i, k)$  subject to (1) and (2).

Suppose  $(j, l)$  chooses a capital stock  $k'_{jl} \neq k_{jl}$ . The resulting market capitalization is denoted  $v'_{jl}$ . Utility maximization yields  $(c'_i, \theta'_i, a'_i, z'_i)$ . From  $v'_{jl} - (p_0 k'_{jl} - R b_{jl}) \leq v_{jl} - (p_0 k_{jl} - R b_{jl})$  and strong monotonicity,  $u_i(c'_i, k) \leq u_i(c_i, k)$ . Suppose  $k'_{jl}$  allows  $i$  to achieve higher overall utility than  $k_{jl}$ :

$$\begin{aligned} & \hat{U}_i(u_i(c'_i, k), \theta'_{i11} v_{11}^{D_{i11}}, \dots, \theta'_{ijl} (v'_{jl})^{D_{ijl}}, \dots, \theta'_{iJLL} v_{JLL}^{D_{iJLL}}, v_M) \\ & > \hat{U}_i(u_i(c_i, k), \theta_{i11} v_{11}^{D_{i11}}, \dots, \theta_{ijl} v_{jl}^{D_{ijl}}, \dots, \theta_{iJLL} v_{JLL}^{D_{iJLL}}, v_M). \end{aligned}$$

First, let  $(j, l) \in \mathcal{J}_i^c$ . Given  $k_{jl}$ , from Lemma 2,  $i$  can achieve  $c_i$  with a share  $\theta'_{ijl} v'_{jl} / v_{jl}$  in  $(j, l)$  and shares  $\theta'_{ij'l'}$  in  $(j', l') \in \mathcal{J}_i^c \setminus \{(j, l)\}$ . Since  $u_i(c'_i, k) \leq u_i(c_i, k)$ , the ensuing utility satisfies

$$\begin{aligned} & \hat{U}_i(u_i(c_i, k), \theta'_{i11} v_{11}^{D_{i11}}, \dots, \theta'_{ijl} (v'_{jl})^{D_{ijl}}, \dots, \theta'_{iJLL} v_{JLL}^{D_{iJLL}}, v_M) \\ & \geq \hat{U}_i(u_i(c'_i, k), \theta'_{i11} v_{11}^{D_{i11}}, \dots, \theta'_{ijl} (v'_{jl})^{D_{ijl}}, \dots, \theta'_{iJLL} v_{JLL}^{D_{iJLL}}, v_M). \end{aligned}$$

Jointly, the two inequalities contradict the fact that  $(c_i, \theta_i, a_i, z_i)$  maximizes  $\hat{U}_i$  given  $k_{jl}$ . If, on the other hand,  $(j, l) \in \mathcal{J}_i$ , then  $i$  can achieve  $c_i$  with shares  $\theta'_{ij'l'}$  in  $(j', l') \in \mathcal{J}_i^c$ , and the same contradiction occurs.  $\square$

**PROOF OF THEOREM 1.** Let  $\mathbf{p}^*$ ,  $\mathbf{v}^*$ ,  $R^*$ , and  $\mathbf{q}^*$  be the same in  $\mathcal{E}_0$  as in  $\mathcal{E}$ . Suppose the state prices  $\mathbf{r}$  are also the same in  $\mathcal{E}_0$  and in  $\mathcal{E}$ . Then  $\sum_{s=1}^S r_s [p_{ls}^* f_{jls}(k_{jl}) - b_{jl}] - (p_0^* k_{jl} - R^* b_{jl})$  is the same in  $\mathcal{E}_0$  as in  $\mathcal{E}$ . SV maximization under competitive expectations implies that the capital stocks  $k_{jl}^*$  and the resulting outputs  $y_{jls} = f_{jls}(k_{jl}^*)$  optimal in  $\mathcal{E}$  maximize SV in  $\mathcal{E}_0$  as well. It follows that the vector of state prices  $\mathbf{r}$  determined by (3)–(5) is in fact the same in  $\mathcal{E}_0$  as in  $\mathcal{E}$ . Furthermore, the consumers’ budget constraints (1) and (2) are also the same in  $\mathcal{E}_0$  as in  $\mathcal{E}$ . From Lemma 3,  $(\mathbf{c}_i^*, \boldsymbol{\theta}_i^*, \mathbf{a}_i^*, \mathbf{z}_i^*)$  maximizes utility in  $\mathcal{E}_0$ . Market clearing for goods, stocks, bonds, and securities in  $\mathcal{E}$  implies market clearing in  $\mathcal{E}_0$ .  $\square$

**PROOF OF THEOREM 2.** Let  $\mathbf{p}^*$ ,  $\mathbf{v}^*$ ,  $R^*$ ,  $\mathbf{q}^*$ , and  $\mathbf{r}$  be the same in  $\mathcal{E}$  as in  $\mathcal{E}_0$ . Then SV maximization yields the same capital stocks  $k_{jl}^*$  and outputs  $y_{jls}$ , so that the state prices determined by (3)–(5) are in fact the same in  $\mathcal{E}$  as in  $\mathcal{E}_0$ , and the budget constraints are also the same. From Lemma 3, for each  $i$ , there exists a portfolio  $(\boldsymbol{\theta}_i^*, \mathbf{a}_i^*, \mathbf{z}_i^*)$  that jointly with  $\mathbf{c}_i^*$  maximizes  $U_i$ .

It remains to show that the set of profiles  $(\mathbf{c}_i^*, \boldsymbol{\theta}_i^*, \mathbf{a}_i^*, \mathbf{z}_i^*)_{i=1}^I$  contains an element that implies asset market clearing. Let  $(\Delta\boldsymbol{\theta}_i, \Delta\mathbf{a}_i, \Delta\mathbf{z}_i) = (\boldsymbol{\theta}_i^*, \mathbf{a}_i^*, \mathbf{z}_i^*) - (\boldsymbol{\theta}_i^{**}, \mathbf{a}_i^{**}, \mathbf{z}_i^{**})$ . For all  $i$ ,  $(\mathbf{c}_i^*, \boldsymbol{\theta}_i^{**}, \mathbf{a}_i^{**}, \mathbf{z}_i^{**})$  satisfies (1) and (2) with equality, as it maximizes utility in  $\mathcal{E}_0$ . The vector  $(\mathbf{c}_i^*, \boldsymbol{\theta}_i^*, \mathbf{a}_i^*, \mathbf{z}_i^*)$  also satisfies (1) and (2) with equality if, and only if,

$$\sum_{l=1}^L \sum_{j=1}^{J_l} (v_{jl}^* \Delta\theta_{ijl} + R^* \Delta a_{ijl}) + \sum_{m=1}^M q_m^* \Delta z_{im} = 0 \tag{A.2}$$

$$\sum_{l=1}^L \sum_{j=1}^{J_l} [(p_{ls}^* y_{jls} - b_{jl}) \Delta\theta_{ijl} + \Delta a_{ijl}] + \sum_{m=1}^M x_{ms} \Delta z_{im} = 0, \quad s = 1, \dots, S. \tag{A.3}$$

Let  $i'$  be a neutral consumer. From Lemma 2, there exist  $(\boldsymbol{\theta}_{i'}^*, \mathbf{a}_{i'}^*, \mathbf{z}_{i'}^*)$  and, hence,  $(\Delta\boldsymbol{\theta}_i, \Delta\mathbf{a}_i, \Delta\mathbf{z}_i)$  such that  $(\mathbf{c}_i^*, \boldsymbol{\theta}_i^*, \mathbf{a}_i^*, \mathbf{z}_i^*)$  satisfies (1) and (2) with equality and, hence, (A.2) and (A.3) for all  $i \neq i'$ . Take these  $(\boldsymbol{\theta}_i^*, \mathbf{a}_i^*, \mathbf{z}_i^*)$  as given. Summing (A.2) and (A.3) over all  $i \neq i'$  yields

$$\sum_{l=1}^L \sum_{j=1}^{J_l} \left( v_{jl}^* \sum_{i \neq i'} \Delta\theta_{ijl} + R^* \sum_{i \neq i'} \Delta a_{ijl} \right) + \sum_{m=1}^M q_m^* \sum_{i \neq i'} \Delta z_{im} = 0 \tag{A.4}$$

$$\sum_{l=1}^L \sum_{j=1}^{J_l} \left[ (p_{ls}^* y_{jls} - b_{jl}) \sum_{i \neq i'} \Delta\theta_{ijl} + \sum_{i \neq i'} \Delta a_{ijl} \right] + \sum_{m=1}^M x_{ms} \sum_{i \neq i'} \Delta z_{im} = 0, \tag{A.5}$$

$s = 1, \dots, S.$

Let

$$(\Delta\boldsymbol{\theta}_{i'}, \Delta\mathbf{a}_{i'}, \Delta\mathbf{z}_{i'}) = \left( - \sum_{i \neq i'} \Delta\boldsymbol{\theta}_i, - \sum_{i \neq i'} \Delta\mathbf{a}_i, - \sum_{i \neq i'} \Delta\mathbf{z}_i \right),$$

so that the asset markets clear. From (A.4) and (A.5),  $(\mathbf{c}_{i'}^*, \boldsymbol{\theta}_{i'}^*, \mathbf{a}_{i'}^*, \mathbf{z}_{i'}^*)$  satisfies (A.2) and (A.3), and, hence, (1) and (2) with equality for  $i = i'$ .  $\square$

**PROOF OF THEOREM 3.** Let  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, k^*)$  be Pareto-optimal in  $\mathcal{E}_0$ . As consumers' utilities do not depend on asset holdings,  $((c_i^*)_{i=1}^I, k^*)$  is Pareto-optimal in  $\tilde{\mathcal{E}}$ . Suppose there are  $t_{jl}$ ,  $T_i$ , and  $\tilde{p}^*$  such that  $((c_i^*)_{i=1}^I, k^*, \tilde{p}^*)$  is an equilibrium of  $\tilde{\mathcal{E}}$ . From equilibrium equivalence in  $\tilde{\mathcal{E}}$  and  $\mathcal{E}_0$ , it follows that, given  $t_{jl}$  and  $T_i$ , there exists  $(p^*, v^*, R^*, q^*)$  such that  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, k^*, p^*, v^*, R^*, q^*)$  is an equilibrium of  $\mathcal{E}_0$ .

Given positive responsiveness, the set of Pareto-optimal allocations of  $\mathcal{E}$  is a subset of the set of Pareto-optimal allocations of  $\mathcal{E}_0$ . To see this, let  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, k^*)$  be Pareto-optimal in  $\mathcal{E}$ . Suppose there is an allocation  $((c_i, \theta_i', a_i', z_i')_{i=1}^I, k)$  that is Pareto-superior to  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, k^*)$  in  $\mathcal{E}_0$ . That implies  $u_i(c_i, k) \geq u_i(c_i^*, k^*)$  for all  $i$  with strict inequality for some  $i$ . By positive responsiveness, there exists  $(\theta_i, a_i, z_i)_{i=1}^I$  such that  $((c_i, \theta_i, a_i, z_i)_{i=1}^I, k)$  is feasible and

$$U_i(u_i(c_i, k), \theta_i, a_i, k, v) \geq U_i(u_i(c_i^*, k^*), \theta_i^{**}, a_i^{**}, k^*, v)$$

for all  $i$  with strict inequality for some  $i$  in  $\mathcal{E}$ . That is, the allocation  $((c_i, \theta_i, a_i, z_i)_{i=1}^I, k)$  is Pareto-superior to  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, k^*)$  in  $\mathcal{E}$ , a contradiction.

Taken together, it follows that given an allocation  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, k^*)$  that is Pareto-optimal in  $\mathcal{E}$ , there is an equilibrium  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, k^*, p^*, v^*, R^*, q^*)$  of  $\mathcal{E}_0$ . From Theorem 2, there is an equilibrium  $((c_i^*, \theta_i^*, a_i^*, z_i^*)_{i=1}^I, k^*, p^*, v^*, R^*, q^*)$  of  $\mathcal{E}$ . From Lemma 2,

$$U_i(u_i(c_i^*, k^*), \theta_i^*, a_i^*, k^*, v) \geq U_i(u_i(c_i^*, k^*), \theta_i^{**}, a_i^{**}, k^*, v) \tag{A.6}$$

for all  $i$ . Suppose the inequality is strict for some  $i$ . Then, starting from  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, k^*)$ , reallocating  $(\Delta\theta_i, \Delta a_i) = (\theta_i^*, a_i^*) - (\theta_i^{**}, a_i^{**})$  from a neutral consumer to  $i$  constitutes a Pareto improvement, a contradiction. So (A.6) holds with equality for all  $i$ . This proves that  $((c_i^*, \theta_i^{**}, a_i^{**}, z_i^{**})_{i=1}^I, k^*, p^*, v^*, R^*, q^*)$  is an equilibrium of  $\mathcal{E}$ .  $\square$

**PROOF OF LEMMA 5.** Substituting for  $c_{i0}$  and  $c_{is}$  from (1) and (2) with equality into  $U_i$ , differentiating with respect to  $k_j$  holding  $e_i(k)$  and  $v_M$  constant, and using multiplicative uncertainty gives

$$\begin{aligned} \frac{dU_i}{dk_j} &= \frac{\partial U_i}{\partial u_i} \left\{ \nu'_i(c_{i0}) \left[ -\bar{\theta}_{ij} - (\theta_{ij} - \bar{\theta}_{ij}) \frac{dv_j}{dk_j} \right] + E[\beta_i \nu'_i(c_i) \theta_{ij} \lambda_j f'_j(k_j)] \right\} e_i \\ &\quad + \frac{\partial U_i}{\partial k_j} + \frac{\partial U_i}{\partial v_j} \frac{dv_j}{dk_j} + \sum_{j'=1}^J \left( \frac{\partial U_i}{\partial u_i} \frac{\partial u_i}{\partial \theta_{ij'}} + \frac{\partial U_i}{\partial \theta_{ij'}} \right) \frac{d\theta_{ij'}}{dk_j} + \frac{\partial U_i}{\partial u_i} \frac{\partial u_i}{\partial z_i} \frac{dz_i}{dk_j}. \end{aligned}$$

From (7) and (8), the final two terms in the sum on the right-hand side vanish in equilibrium. Using

$$E[\beta_i \nu'_i(c_i) \theta_{ij} \lambda_j f'_j(k_j)] = \nu'_i(c_{i0}) \theta_{ij} \left[ 1 - \frac{1}{\nu'_i(c_{i0}) e_{iv_j}} \frac{\partial U_i / \partial \theta_{ij}}{\partial U_i / \partial u_i} \right] \frac{dv_j}{dk_j}$$

(from (9) and (10) and the fact that the denominator of the fraction in (10) is nonrandom) and simplifying terms yields (12).  $\square$

LEMMA 6. For all  $i$ , suppose for each firm  $j' \in \mathcal{J}_i^c$ , there is a firm  $j'' \in \mathcal{J}_i$  with  $\lambda_{j''s} = \lambda_{j's}$  for all  $s$ . Let  $(c_i, \theta_i, z_i)$  satisfy (1) and (2), and let  $\theta'_{ij}$  be given for  $j \in \mathcal{J}_i^c$ . Then there exists  $\theta'_{ij}$  for  $j \in \mathcal{J}_i$  such that  $(c_i, \theta'_i, z_i)$  satisfies (1) and (2).

PROOF. Given  $j' \in \mathcal{J}_i^c$ , consider the set  $\mathcal{J}'$  of all  $j$  with identical productivity shocks. Pick any  $j'' \in \mathcal{J}' \cap \mathcal{J}_i (\neq \emptyset)$ . Define

$$\mu_j = \frac{f_j(k_j)}{f_{j''}(k_{j''})} = \frac{\lambda_{js}f_j(k_j)}{\lambda_{j''s}f_{j''}(k_{j''})} = \frac{y_{js}}{y_{j''s}}, \quad j \in \mathcal{J}'.$$

Consider a neutral consumer  $i'$ . From (9) with  $\tilde{m}_{i'j} = m_{i'}$  for all  $j$ ,

$$\mu_j = \frac{E(m_{i'}\mu_j y_{j''})}{E(m_{i'} y_{j''})} = \frac{E(m_{i'} y_j)}{E(m_{i'} y_{j''})} = \frac{v_j}{v_{j''}}, \quad j \in \mathcal{J}'.$$

The set  $\mathcal{J}$  can be partitioned into subsets  $\mathcal{J}'$  of firms with identical productivity shocks. If  $\mathcal{J}' \cap \mathcal{J}_i^c = \emptyset$ , let  $\theta'_{ij} = \theta_{ij}$  for all  $j \in \mathcal{J}'$ . For  $\mathcal{J}'$  with  $\mathcal{J}' \cap \mathcal{J}_i^c \neq \emptyset$ , there exists  $\theta'_{ij}$  for  $j \in \mathcal{J}' \cap \mathcal{J}_i$  such that

$$\sum_{j \in \mathcal{J}'} \theta'_{ij} \mu_j = \sum_{j \in \mathcal{J}'} \theta_{ij} \mu_j.$$

Pick any  $j'' \in \mathcal{J}' \cap \mathcal{J}_i$ . Then

$$\sum_{j \in \mathcal{J}'} \theta'_{ij} v_j = \sum_{j \in \mathcal{J}'} \theta'_{ij} \mu_j v_{j''} = v_{j''} \sum_{j \in \mathcal{J}'} \theta'_{ij} \mu_j = v_{j''} \sum_{j \in \mathcal{J}'} \theta_{ij} \mu_j = \sum_{j \in \mathcal{J}'} \theta_{ij} \mu_j v_{j''} = \sum_{j \in \mathcal{J}'} \theta_{ij} v_j$$

and, analogously,

$$\sum_{j \in \mathcal{J}'} \theta'_{ij} y_{js} = \sum_{j \in \mathcal{J}'} \theta_{ij} y_{js}.$$

It follows that if  $(c_i, \theta_i, z_i)$  satisfies (1) and (2), so does  $(c_i, \theta'_i, z_i)$ . □

PROOF OF THEOREM 4. From Lemma 6,  $\partial U_i / \partial \theta_{ij} = 0$  for  $j \in \mathcal{J}_i^c$  at an equilibrium of  $\mathcal{X}$ , for otherwise there exists  $\theta'_i$  such that  $(c_i^*, \theta'_i, z_i^*)$  satisfies (1) and (2) and  $U_i(u_i(c_i^*, k^*), \theta'_i, k^*, v^*) > U_i(u_i(c_i^*, k^*), \theta_i^*, k^*, v^*)$ , a contradiction. From (10),  $\tilde{m}_{ij} = m_i$  for all  $j \in \mathcal{J}$ . Using Lemma 6 instead of Lemma 2, the assertions of Lemma 3 follow from the same arguments as in the proof above.

Let  $v^*$ ,  $q^*$ , and  $m_i$  ( $i = 1, \dots, I$ ) be the same in  $\mathcal{X}_0$  as in  $\mathcal{X}$ . Then SV maximization implies that  $k_j^*$ ,  $y_{js}$  ( $j = 1, \dots, J$ ) and, hence, the consumers' budget constraints are the same in  $\mathcal{X}_0$  as in  $\mathcal{X}$ . From Lemma 3,  $(c_i^*, \theta_i^*, z_i^*)$  maximizes utility in  $\mathcal{X}_0$ . It follows that the  $m_i$ s are in fact the same as in  $\mathcal{X}$ . Market clearing for goods, stocks, and the safe security in  $\mathcal{X}$  implies market clearing in  $\mathcal{X}_0$ , so an equilibrium of  $\mathcal{X}$  is also an equilibrium of  $\mathcal{X}_0$ .

As for the converse, let  $v^*$ ,  $q^*$ , and  $m_i$  ( $i = 1, \dots, I$ ) be the same in  $\mathcal{X}$  as in  $\mathcal{X}_0$ . Then SV maximization yields the same  $k_j^*$ ,  $y_j$  ( $j = 1, \dots, J$ ), and budget constraints. From Lemma 3, there exists a portfolio  $(\theta_i^*, z_i^*)$  that jointly with  $c_i^*$  maximizes utility for  $i$ . The

SDFs are in fact the same as in  $\mathcal{X}_0$ . The same arguments as in the proof of Theorem 2 establish that the set of profiles  $(c_i^*, \theta_i^*, z_i^*)_{i=1}^I$  contains an element that is compatible with asset market clearing.  $\square$

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