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Agendas in legislative decision-making

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Despite the wide variety of agendas used in legislative settings, the literature on sophisticated voting has focused on two formats: the so-called Euro–Latin and Anglo–American agendas. In the current paper, I introduce a broad class of agendas whose defining structural features—history-independence and persistence—are common in legislative settings. I then characterize the social choice rules implemented by sophisticated voting on agendas with these two features. I also characterize the rules implemented by more specialized formats (called priority agendas and convex agendas) whose structure is closely related to the prevailing rules for order-of-voting used by legislatures. These results establish a clear connection between structure and outcomes for a wide range of legislative agendas.

Keywords. Majority voting, sophisticated voting, agendas, committees, implementation.

JEL classification. C72, D02, D71, D72.

1. Introduction

An agenda is a mechanism for collective decision-making that specifies a sequence of "yea" (yes) or "nay" (no) questions. Each successive question, to be decided by majority vote, forecloses some options until only a single alternative remains. In legislative settings, agendas are used to decide almost every issue. Dating back to the work of Black (1948, 1958) and Farquharson (1969), the goal has been to understand how the structure of the agenda shapes the legislative outcome when legislators behave in a *sophisticated* (or strategic) fashion.

Since Farquharson, the literature has focused on two formats, called *Euro–Latin* and *Anglo–American* agendas, that arise in legislative settings. Figure 1 illustrates both formats for the case of three alternatives.[1] On a Euro–Latin agenda (left), the voters consider one alternative at a time, ultimately selecting the first to be approved by a majority.

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[1] The names for these formats are not standard. Some follow Farquharson, who referred to Euro–Latin agendas as *successive agendas* and to Anglo–American agendas as *amendment agendas*. Black referred to the latter as *ordinary committee procedure*.

[Correction added on 9 March 2023, after first online publication: References to the online appendix were updated.]

Q1. Shall x_1 be approved?
Q2. If not, shall x_2 be approved?

Q1. Shall x_1 be selected over x_2?
Q2. Shall the winner of Q1 be selected over x_3?

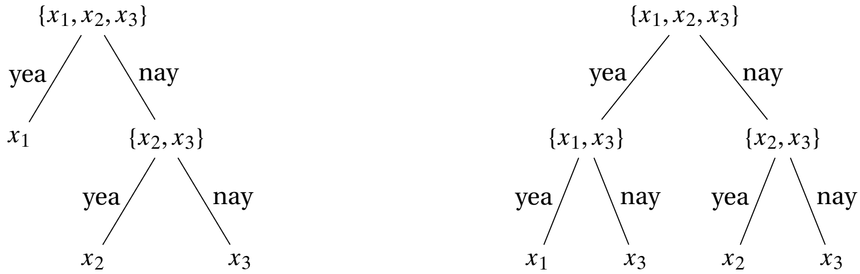


Figure 1. Left: A Euro–Latin agenda on three alternatives. Right: An Anglo–American agenda on three alternatives.

On an Anglo–American agenda (right), the voters compare two alternatives at a time, with the majority winner progressing to the next stage and the loser being eliminated. Ultimately, the alternative selected is the one that survives this sequential process of pairwise comparison.

While Farquharson's analysis did not go beyond agendas with three alternatives, later work characterized the sophisticated voting outcomes for Euro–Latin and Anglo–American agendas of any size.[2] Miller (1977, 1980) showed that Euro–Latin agendas lead to outcomes within the *top cycle* of the majority relation while Banks (1985) showed that Anglo–American agendas lead to outcomes within a subset of the more restrictive *uncovered set*.[3] More recently, Apesteguia et al. (2014) showed how the outcome implemented by each format co-varies with the profile of voter preferences and the set of feasible alternatives.

In the literature, the focus on Euro–Latin and Anglo–American agendas is usually justified by the assertion that these formats approximate the legislative process in civil law and common law jurisdictions (Miller 1977, p. 780). Ordeshook and Schwartz (1987) raised serious doubts about this view. Taking into account a variety of real-world examples, they concluded that legislative agendas regularly depart from the Euro–Latin and Anglo–American formats. Their conclusion is supported by a variety of other surveys and case studies (Bach 1983, Sullivan 1984, Krehbiel and Rivers 1990, Calvert and Fenno

[2]For real-world examples of both formats with more than three alternatives, see Ladha (1994, p. 60) and Senti (1998, p. 16).

[3]In the literature, this set is commonly known as the *Banks set*. The top cycle, uncovered set, and Banks set are examples of tournament solutions. For an overview of the vast literature on this topic, see Laslier (1997) or Brandt et al. (2016).

1994, Miller 1995, Volden 1998, Schwartz 2008).[4] The practical implication is that legislative agendas can lead to policy outcomes different from those associated with Euro–Latin and Anglo–American agendas. This is troubling. Agendas play a crucial role in legislative decision-making. Without a clear understanding of what outcomes they induce, one cannot begin to address some of the most basic questions in political economy or public economics.

In the current paper, I characterize the outcomes implemented by a broad range of legislative agendas. To do so, I abstract away from the details of legislative procedure and focus on the emergent "procedural structure" (Sullivan 1984). The simple agendas that I define are distinguished by two structural features: *history-independence*, which stipulates that the question posed at any stage can depend only on the alternatives not yet eliminated from consideration, and *persistence*, which stipulates that "safe" alternatives (that do not risk elimination) at a given stage can be tested only after all questions related to the "contested" alternatives (that do risk elimination) have been settled. Both features are common in practice. Only agendas that include procedural questions seem to violate history-independence; and only agendas that provide for parallel consideration of a bill and a substitute bill (like those sometimes used by the U.S. Congress) seem to violate persistence.

My main contribution is to determine what decision rules are implemented by simple agendas: As one varies the profile of voter preferences or the feasible set of alternatives, how does the sophisticated voting outcome change? Theorem 1 shows that two conditions, called *issue splitting* and the *independence of losing alternatives*, characterize implementation by simple agenda. The first condition, which weakens Plott's (1973) path independence, stipulates that the outcome can be determined by splitting the issue into two subsets of alternatives. In turn, the second condition, which weakens the independence of irrelevant alternatives (Chernoff 1954, Radner and Marschak 1954) stipulates that the removal of an alternative without any majority appeal (called the *Condorcet loser*) cannot affect the outcome.

I also characterize implementation by some more specialized formats that arise in legislative settings. Theorem 2 shows that Euro–Latin and Anglo–American agendas differ from other simple agendas by the way that they marginalize or discriminate against certain alternatives. For Anglo–American agendas, every issue includes one alternative that is chosen only when it has majority appeal over every alternative (i.e., it is the *Condorcet winner*). For Euro–Latin agendas, every issue includes two such alternatives. By highlighting the fundamental similarities between the two formats, these characterizations stand in contrast to the characterizations given by Apesteguia et al. (2014), which instead highlight some key differences.[5]

The important role played by marginalization extends to formats whose structure is related to the order-of-voting rules typically used in legislative settings—specifically

[4]The notable outlier is Rasch (2000, p. 8), who concludes that Euro–Latin and Anglo–American agendas are dominant. However, his study was "not based on a close inspection of voting practices" and did not show "how stringently the two procedures are applied in day-to–day proceedings." For further discussion of Rasch's methodology, see Schwartz (2008, p. 368).

[5]See the Online Appendix (https://econtheory.org/supp/2618/supplement.pdf).

priority agendas, whose sequential structure is related to the rule of precedence used by Anglo–American jurisdictions, and *convex agendas*, whose structure is related to the extremeness rule favored by Euro–Latin jurisdictions. (While convex agendas were previously considered by Kleiner and Moldovanu (2017), the current paper is the first to consider priority agendas.[6]) Theorem 3 shows that priority agendas are distinguished from other simple agendas by the fact that they marginalize some alternative for every issue. In turn, Theorem 4 shows that convex agendas are distinguished by the fact that they marginalize one alternative only to facilitate the choice of another.

I conclude the paper by highlighting the connection between simple agendas and May's (1952) desiderata for voting rules. Theorem 5 shows that every rule implemented by simple agenda is *positively responsive*: the outcome remains unchanged when it improves in terms of voter preferences. Since agendas also treat voters symmetrically (anonymously), every rule implemented by simple agenda satisfies two of May's criteria. However, his third criterion, which requires alternatives to be treated symmetrically (neutrally), cannot be satisfied by any agenda containing more than two alternatives.[7] Theorems 2–4 sharpen this observation by characterizing ways that a range of agenda formats used in legislative settings treat alternatives asymmetrically.

Related Literature: The main strand of the sophisticated voting literature has focused on a narrow range of agendas. Indeed, almost all of the papers (including, e.g., recent work by Gershkov et al. 2017 or Barberà and Gerber 2017) are concerned with Euro–Latin and Anglo–American agendas specifically. Only a handful of papers consider other agenda formats (Banks 1989, Coughlan and Breton 1999, Fischer et al. 2011, Iglesias et al. 2014, Schwartz 2008). A much smaller strand of the literature is concerned with general conditions that are necessary (McKelvey and Niemi 1978, Moulin 1986, Srivastava and Trick 1996) or necessary and sufficient (Horan 2013) for implementation by agenda. However, the results in these papers are largely nonconstructive, so they say little about what kinds of outcomes can be achieved with specific agenda formats.

The current paper bridges the gap between these two strands of the literature by characterizing the relationship between form and function for a wide range of agendas used in legislative settings. While the prior work on this issue is quite limited, two papers are worth mentioning. The first is the paper by Ordeshook and Schwartz (1987), who propose a taxonomy of agenda features (which was later extended by Miller 1995).[8] While I do not rely on their taxonomy, my work is motivated by the same desire to abstract away from institutional details and focus on the structural features of agendas that arise in legislative settings. The other relevant paper is Apesteguia et al. (2014), who introduce the decision rule framework that I adopt in the current paper. From an implementation standpoint, their framework is unusual because it allows the voter preferences *and* the set of alternatives to vary. While this extra degree of freedom does not

[6]Convex agendas are also studied by Gershkov et al. (2019) and Kleiner and Moldovanu (2018).

[7]More generally, it is possible for a single-winner voting rule to treat both the voters and the alternatives symmetrically only under strong restrictions on the number of voters and alternatives. For further details, see Moulin (1988, p. 253).

[8]While Ordeshook and Schwartz also characterize the range of outcomes associated with several agenda formats (à la Banks and Miller), they do not characterize the outcomes implemented by any of these formats (à la Apesteguia et al.).

affect the scope of implementable rules (see Remark 1), it does simplify the interpretation of the conditions for implementation. As a collateral benefit, the decision rule framework is also quite natural in applications.

2. Simple agendas

In this section, I define agendas in general terms before describing the two structural features that define simple agendas. I conclude by presenting several examples of simple agendas that arise in legislative settings.

2.1 *Agendas*

The universe of social *alternatives* is a finite set X such that $|X| \geq 2$ and an *issue* A is a nonempty subset of X. The collection of all issues A such that $|A| \geq 2$ is denoted by $\mathbf{X}$.

Definition 1. An *agenda* $\mathbb{T}_A$ on $A \in \mathbf{X}$ is a *finite binary tree* such that the following conditions hold:

(i) Each *terminal node* is *labeled* by (a set consisting of) one alternative in A.

(ii) Each alternative in A labels one or more terminal nodes.

(iii) Each *nonterminal node* is labeled by the set of alternatives that label its two *successors*.

To simplify, I omit the subscript for an agenda on the universal issue X (writing $\mathbb{T}$ instead of $\mathbb{T}_X$). The Appendix contains a primer of the graph theory terminology used above and elsewhere in the paper.

An agenda defines a *game tree,* where each *outcome* (terminal node) represents a social alternative and each *stage game* (nonterminal node) represents a simultaneous majority vote between the two subgames that follow it. In legislative settings, every vote addresses a yea or nay question. Features (i) and (ii) of Definition 1 ensure that every sequence of questions leads to an outcome and every outcome results from some sequence of questions. Feature (iii) is a labeling convention: it identifies each node with the outcomes that are *reachable* from it.[9] It is worth emphasizing that no aspect of my analysis depends on this convention. It serves only to simplify the definition of the agenda classes that I consider.

In the sequel, I depart from Figure 1 and omit the edge labels from agenda diagrams. Instead, I follow the convention that, at any nonterminal node, the left subgame represents majority support for yea, while the right subgame represents majority support for nay. As in Figure 1, I also omit the set brackets for the terminal nodes. In fact, I omit the brackets for singleton sets whenever doing so causes no confusion.

[9]This convention follows the Farquharson–Miller approach rather than the Ordeshook–Schwartz approach (see Schwartz 2008).

2.2 *Simple agendas*

Two structural features define the agendas that I consider in the sequel. The first specifies that for all nodes leading to the same potential outcomes, the agenda poses the same question. Given an agenda, let $\ell(q)$ denote the label of a node q, and let q_y (yea) and q_n (nay) denote the two successors of a nonterminal node q. Then an agenda is *history-independent* if, for all pairs of nonterminal nodes q and $\tilde{q}$ such that $\ell(q) = \ell(\tilde{q})$,

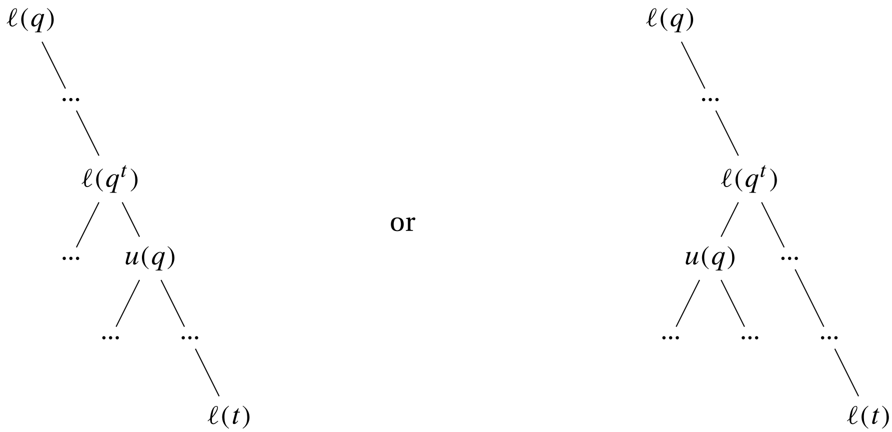
$$\{\ell(q_y), \ell(q_n)\} = \{\ell(\tilde{q}_y), \ell(\tilde{q}_n)\}.$$

In other words, the structure of the agenda below q depends on the set of reachable alternatives $\ell(q)$ but not the sequence of questions leading up to q. As a result, there is little confusion in referring to a node q of a history-independent agenda by its label $\ell(q)$. To simplify, I follow this convention in the sequel.[10]

The second feature specifies that the agenda cannot address new issues until contested issues have been resolved. For a node q, let $u(q) \equiv \ell(q_y) \cap \ell(q_n)$ denote the set of *uncontested outcomes* that face no risk of elimination at q. Conversely, let $c(q) \equiv \ell(q) \setminus u(q)$ denote the set of *contested outcomes* that do risk elimination at q.[11] Then, an agenda is *persistent* if, for every node q such that $u(q) \neq \varnothing$ and each terminal node t below q, there is a nonterminal node q^t between q and t (potentially q itself) such that

$$u(q) \in \{\ell(q^t_y), \ell(q^t_n)\}.$$

The two possibilities associated with this condition are illustrated below. Either some node between q and t is labeled $u(q)$ (left) or the sibling of some node between q and t is labeled $u(q)$ (right):



The idea is that one of the answers to question q^t resolves the "live" issue $c(q)$ contested at q. From that point on, the agenda contests only alternatives associated with the "new" issue $u(q)$. By definition, none of the previous questions (between q and q^t) contests alternatives in $u(q)$ (see Remark 5 of the Appendix).

[10] See, in particular, Definitions 6 and 7 below.

[11] As a matter of convention, let $u(t) \equiv \varnothing$ and $c(t) \equiv \ell(t)$ for every terminal node t.

Definition 2. An agenda is *simple* if it is history-independent and persistent.

My focus on simple agendas is motivated by the prevalence of history-independence and persistence among legislative agendas. Below, I give some examples of agendas, drawn from a range of different legislative settings, that exhibit these two features. Before turning to these examples, it is worth giving a broader sense of the basis for history-independence and persistence in the rules of legislative procedure.

History-independence relates to the fact that legislative rules for agenda building are generally prospective: starting from any point, the prescribed structure of the agenda depends only on the set of pending proposals. The only notable exception relates to procedural motions that do not necessarily eliminate any outcomes. In a *division of the question* motion, for instance, the issue is whether each part of the bill should be put to vote separately.[12] If the division motion is defeated, then the same set of outcomes remains feasible. Since a division motion cannot be reconsidered, this introduces history-dependence into the agenda.

Persistence can, in part, be explained by the admissibility rules imposed in legislative settings (Sullivan 1984, p. 35). Many jurisdictions require that a new proposal must be "in order" before it can be introduced for debate. Some jurisdictions also require that a new proposal must be "germane" to one of the pending proposals.[13] Broadly speaking, these requirements ensure that new proposals are introduced (and, consequently, voted on) only after a consensus on existing proposals has been sufficiently established.

To ensure that legislative agendas are persistent, the rules for order-of-voting on proposals play an even more significant role. Most legislatures follow one of two schemes: the formalistic *rule of precedence* favored by Anglo–American jurisdictions; or the substantive *extremeness rule* favored by Euro–Latin jurisdictions (see Rasch 2000, p. 15). As I explain in Section 4, both schemes tend to produce persistent agendas.

2.3 *Examples*

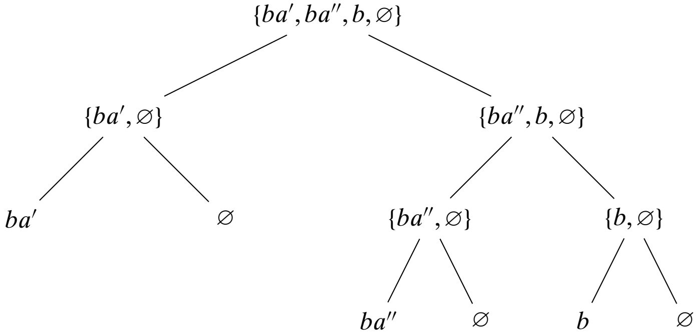
Example 1. When two amendments are proposed to "perfect" a bill, the rule of precedence used by U.S. Congress requires an up-or-down vote on the first amendment before the second can be recognized. If the amendments are incompatible (in the sense that they propose conflicting modifications to the bill), then the second amendment cannot be recognized if the first is adopted. This leads to the following agenda.[14]

[12] See, e.g., House Rule XVI(5). For an agenda that includes such a division of a question, see Schwartz (2018, p. 97).

[13] See, e.g., House Rule XVI(7). In contrast, the Senate Rules do not impose a germaneness requirement (Riddick 1992, p. 62).

[14] Figure 5 of Ordeshook and Schwartz (1987) shows a similar agenda that can arise in Congress.

Q1. *Shall the first amendment a' to the bill b be adopted?*
Q2. *If not, shall the second amendment a'' to bill b be adopted?*
Q3. *Shall the perfected version of the bill replace the status quo legislation $\varnothing$?*

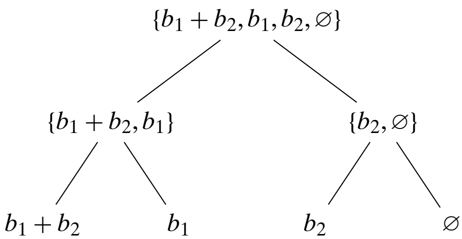


◊

This agenda, which combines elements of Euro–Latin and Anglo–American agendas, is simple.[15] Clearly, it is history-independent: each node has a unique label. It is also persistent: the only alternative uncontested at any node is the status quo $\varnothing$, which is ultimately contested by the final question.

Example 2. In many legislative settings, it is conventional to consider the sections of a (potentially lengthy) proposal "in seriatim" (see Robert 2011, pp. 276–278). If the proposal consists of new bylaws, for instance, then the practice is to vote on the bylaws one at a time.[16] The agenda depicted below serves to illustrate.

Q1. *Shall the first bylaw b_1 be adopted?*
Q2. *Shall the second bylaw b_2 be adopted?*



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This is an example of a *knockout agenda*, so called because each alternative appears at only one terminal node (and can, thus, be "knocked out" by any vote). Since the label

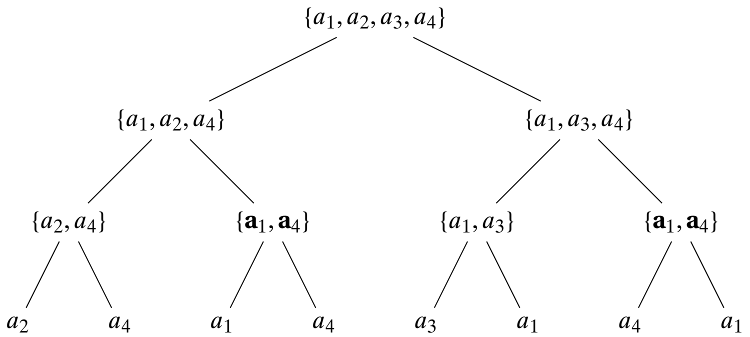
[15]The procedure for perfecting the bill is Euro–Latin: first, the question is whether a' should be approved and, if not, whether a'' should be approved. However, the procedure for adopting the bill is Anglo–American: the last question always pits the perfected bill against the status quo, which is a standard feature in many Anglo–American jurisdictions (see Rasch 2000, Table 2).

[16]Plott and Levine (1978) call this type of agenda a *bill-by-bill agenda*.

of each node is unique, a knockout agenda must be history-independent. Since there are no uncontested alternatives at any node, it must also be persistent.

Example 3. The process for deciding among conflicting floor proposals in the Swiss parliament requires a sequence of elimination votes "first on the proposals that differ the *least* from each other in content, working through the proposals until those that differ the *most* are reached."[17] To illustrate, suppose that four amendments $a_1, \ldots, a_4$ (ordered in terms of similarity) are proposed, with the middle two amendments being most similar.

Q1. *Shall amendment a_2 be adopted over the next most similar amendment a_3?*
Q2. *Shall the winner of Q1 be adopted over the next most similar amendment?*
Q3. *Shall the winner of Q2 be adopted over the remaining amendment?*



$\diamond$

This agenda is history-independent: the only nodes that share the same label occur in the last stage where there is no flexibility about the question to be asked. It is also persistent. To see this, first consider the alternatives $\{a_1, a_4\}$ that are uncontested at the root. Since this set labels a successor of both nodes at the second stage (in bold), the agenda is persistent at the root. The same is true for each node in the second stage. The uncontested alternative a_4 at the left-hand node labels a terminal node below both successors of that node. Likewise, a_1 labels a terminal node below both successors of the right-hand node.

3. Implementation by simple agenda

In this section, I define implementation by agenda before focusing the analysis on simple agendas—showing that two natural conditions characterize the decision rules implemented by agendas in this class.

[17] See Article 79.2 of the Loi sur le Parlement.

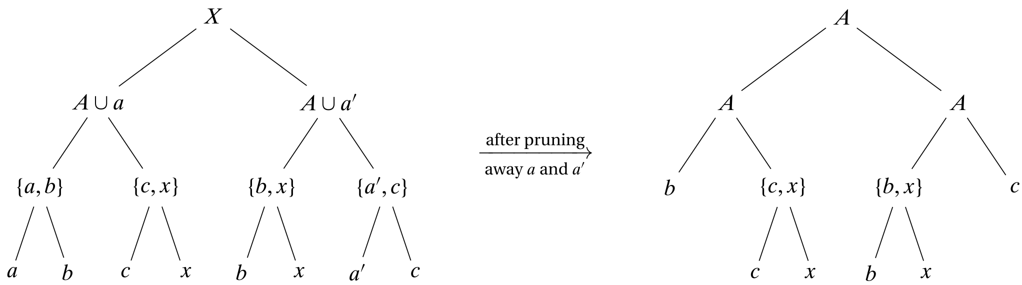


FIGURE 2. Left: An agenda $\mathbb{T}$ on X. Right: The pruned agenda $\mathbb{T}|_A$ on $A \subset X$.

3.1 *Implementation*

Let N denote the finite set of voters. For simplicity, suppose that $|N|$ is odd and $|N| \geq |X|$.[18] A *profile* of voter preferences $P \equiv (P_i)_{i \in N}$ is an $|N|$-tuple of linear orderings P_i over X. A *decision problem* is pair (P, A). Letting $\mathbf{P}$ denote the collection of all profiles, a *decision rule* is a mapping $v : \mathbf{P} \times \mathbf{X} \to X$ such that $v(P, A) \in A$ for all $(P, A) \in \mathbf{P} \times \mathbf{X}$. Thus, v prescribes an outcome for all combinations of issues and voter preferences.

Since my focus is the implementation of decision rules, it is necessary to adapt an agenda $\mathbb{T}$ on the universal issue X to every sub-issue $A \subset X$. The most natural way to do this is to remove all of the terminal nodes labeled by infeasible alternatives (see Xu and Zhou 2007, Horan 2011, Bossert and Sprumont 2013).

DEFINITION 3. Given an agenda $\mathbb{T}$ on X, the *pruned agenda* $\mathbb{T}|_A$ on $A \in \mathbf{X}$ may be obtained as follows:

(i) First, remove every terminal node of $\mathbb{T}$ that is labeled by an alternative $x \in X \setminus A$.

(ii) Then remove every node that has one successor, connecting its successor to its predecessor.

(iii) Finally, relabel every nonterminal node of the resulting tree to conform with Definition 1.

Like a single-elimination sports competition, the idea is that the infeasible alternatives in $X \setminus A$ "forfeit" their place without otherwise modifying the structure of the agenda. To illustrate, consider the agenda $\mathbb{T}$ on $X = \{a, a', b, c, x\}$ and the pruned agenda $\mathbb{T}|_A$ on the subset $A = \{b, c, x\}$ shown in Figure 2.

Given an agenda $\mathbb{T}$ on X, each pair $(\mathbb{T}|_A; P)$ defines a complete information extensive-form game with voter preferences given by P and outcomes in A. The natural solution concept for this type of game is *sophisticated voting* (Farquharson 1969), which is based on the idea that voters are forward-looking. As it turns out, the unique sophisticated voting equilibrium outcome depends only on the preference of the majority.

[18]The assumption that $|N|$ is odd ensures that the majority relation is total and asymmetric. In turn, $|N| \geq |X|$ ensures that every majority relation results from some preference profile. (See footnote 19 and the preceding discussion.)

For $P \in \mathbf{P}$, the *majority relation* M_P between alternatives $x, y \in X$ is defined by

$$xM_P y \quad \text{if} \quad \left|\{i \in N : xP_i y\}\right| > |N|/2.$$

Since $|N|$ is odd, M_P is a total (for distinct $x, y \in X$, $xM_P y$ or $yM_P x$) and asymmetric (for $x, y \in X$, not $xM_P y$ and $yM_P x$) binary relation on X. A relation with these features is usually called a *tournament*.[19]

In any vote at the final stage of an agenda T_A, every voter has a weakly dominant strategy to endorse her preferred alternative. So in any vote at this stage, M_P determines which alternative wins a majority. When deciding how to vote at the penultimate stage, the forward-looking voters discount the alternatives that will lose at the final stage, and again perceive the vote as a choice between two alternatives. So M_P also determines which option (yea or nay) wins a majority at this stage. By extending this reasoning back to the root of the agenda, one can use the majority relation M_P to determine the sophisticated voting outcome of the game $(\mathrm{T}_A; P)$. As shown by McKelvey and Niemi (1978), this "backward induction" reasoning is equivalent to finding the *dominance solvable* (Moulin 1979) outcome of the game $(\mathrm{T}_A; P)$.[20] To formalize:

Definition 4. For a game $(\mathrm{T}_A; P)$, the *sophisticated voting outcome* $\mathrm{SVO}[\mathrm{T}_A; P]$ is defined as follows:

(i) First, define $\mathrm{T}^P_A(1) \equiv \mathrm{T}_A$ to be the original agenda.

(ii) Then, for each $j > 1$, define the agenda $\mathrm{T}^P_A(j+1)$ from $\mathrm{T}^P_A(j)$ as follows:

- In each terminal subgame of $\mathrm{T}^P_A(j)$, prune away the loser according to M_P.
- Relabel the nonterminal nodes of the resulting binary tree to conform with Definition 1.

(iii) Finally, define $\mathrm{SVO}[\mathrm{T}_A; P] \equiv \mathrm{T}^P_A(K)$, where K is the smallest j such that $\mathrm{T}^P_A(j) = \mathrm{T}^P_A(j+1)$.

Having defined the solution concept, agenda implementation may then be defined as follows.

Definition 5. A decision rule v is *implementable by agenda* if there exists an agenda T such that

$$v(P, A) = \mathrm{SVO}[\mathrm{T}|_A; P]$$

[19] McGarvey (1953) was the first to consider the problem of finding the minimum number of voters $|N|$ such that $\{M_P : P \in \mathbf{P}\}$ coincides with the set of *all* tournaments on X. He showed that $|N| \geq |X|^2 - |X|$ is sufficient. Stearns (1959) tightened the bound to $|N| \geq |X| + 2$ and Fiol (1992) later improved Stearns' bound to $|N| \geq |X| - \lfloor \log_2 |X| \rfloor + 1$.

[20] The quotation marks (which I continue to use in the sequel) serve as a reminder that this is not technically backward induction. The problem is the absence of full information: each stage game involves simultaneous choices by all voters.

for every decision problem $(P, A) \in \mathbf{P} \times \mathbf{X}$. In that case, the agenda $\mathbb{T}$ *implements* the decision rule v.

Despite appearances, this formulation is no more general than the usual notion of implementation with a fixed issue X. To see this, consider a decision problem (P, A) and let P^A denote a profile that coincides with P on A except that in each voter preference, the alternatives in $X \setminus A$ are placed (in a fixed order) below every alternative in A. This demotion is equivalent to pruning away the alternatives in $X \setminus A$.

Remark 1. If a decision rule v is implementable by agenda, then $v(P, A) = v(P^A, X)$ for all $(P, A) \in \mathbf{P} \times \mathbf{X}$.

In other words, the sub-issues actually play no role in the analysis of agenda implementation. Once the agenda-setter decides what to implement for the grand issue X, the outcomes for all sub-issues $A \subset X$ are determined.

3.2 *Characterization*

Two conditions are necessary and sufficient for implementation by simple agenda. The first stipulates that the outcome of any decision problem can be determined by "splitting up" the issue into two sub-issues. For a decision rule v, a pair of issues $(B, C) \subseteq X \times X$ defines a *v-splitting* of the issue $A \in \mathbf{X}$ if

(i) $B \cup C = A$ so that B and C form an *exact bi-cover* of A

(ii) $B \cap C \neq B, C$ so that B and C are *nonnested*

(iii) $v(P, A) = v(P, \{v(P, B), v(P, C)\})$ for every profile $P \in \mathbf{P}$.

The first condition stipulates that every issue can be split up in this way.

Issue splitting (IS). *For every issue $A \in \mathbf{X}$, there is a v-splitting.*

While similar in spirit to Plott's (1973) path independence, this condition is weaker. Indeed, Plott's condition imposes the requirement that the equality in (iii) must hold for *all* pairs (B, C) that by (i) form an exact bi-cover of A even if B and C are nested (which conflicts with (ii) above). Issue splitting also weakens Apesteguia et al.'s (2014) *division consistency* (called *weak separability* by Xu and Zhou 2007).[21] The major difference is that (ii) does not require the two sub-issues B and C to be disjoint. Another difference is that (iii) does not impose any form of consistency between the splitting of the issue A and the splitting of its sub-issues $D \subset A$.

The second condition states that the outcome is not affected by the presence of an unappealing alternative. Specifically, alternative $a \in A$ is the *Condorcet loser* for a decision problem $(P, A) \in \mathbf{P} \times \mathbf{X}$ if xM_Pa for all $x \in A \setminus a$. A decision rule v is *independent*

[21] Formally, this condition may be stated as follows: For every issue $A \in \mathbf{X}$, there exists a pair $(B, C) \in X \times X$ such that (i') $B \cup C = A$, (ii') $B \cap C = \varnothing$, and (iii') $v(P, D) = v(P, \{v(P, B \cap D), v(P, C \cap D)\})$ for each profile $P \in \mathbf{P}$ and each $D \subseteq A$.

of the losers for the issue $A \in \mathbf{X}$ if for all $P \in \mathbf{P}$ and $a \in A$, $v(P, A) = v(P, A \setminus a)$ if a is the Condorcet loser for (P, A).[22]

Independence of losing alternatives[23] (ILA). *For every issue $A \in \mathbf{X}$, v is independent of the losers.*

By comparison, the well known independence of irrelevant alternatives (IIA) imposes the much stronger requirement that the removal of *any* unchosen alternative keeps the outcome unchanged.[24]

These two conditions characterize the decision rules that can be implemented by simple agenda. What is more, the simple agenda that implements the rule is unique and its structure can be inferred from the outcomes of v for profiles, called *Condorcet triples*, where the majority relation forms a cycle on three alternatives.

Theorem 1 (Necessary and sufficient conditions). *A decision rule v is implementable by simple agenda if and only if it satisfies IS (issue splitting) and ILA (independence of losing alternatives). (Uniqueness) For every decision rule v that satisfies these two properties, there is a unique simple agenda $\mathbb{S}^v$ that implements v. What is more, the structure of the agenda $\mathbb{S}^v$ is fully determined by the outcomes of v on Condorcet triples.*

ILA is necessary for implementation by all agendas (not just simple agendas).[25] This is because "backward induction" is unaffected by pruning away the Condorcet loser (see Remark 1 above). However, as the next example shows, IS is not generally necessary. To simplify, let P_{xyz} denote a Condorcet triple where x is majority preferred to y, y is majority preferred to z, and z is majority preferred to x.

Example 4. The rule implemented by the agenda $\mathbb{T}$ in Figure 2 violates IS. For the Condorcet triple P_{xbc}, a majority of voters prefer x to the alternative b chosen for (P_{xbc}, A). As a result, x cannot be paired with b in the splitting of A. By similar reasoning about (P_{xcb}, A), x cannot be paired with c. This leaves $(x, \{b, c\})$ as the only potential splitting of A. However, this requires x to be chosen for (P_{xbc}, A) and (P_{xcb}, A). ◊

For the sufficiency and uniqueness portions of Theorem 1, the key is to show that IS and ILA imply that every issue has a unique splitting (Claim 5 of the Appendix). One can then define an agenda $\mathbb{S}^v$ that subdivides the issue $\ell(q)$ at any node q according to its

[22]To ensure that this identity is well defined in the case where $|A| = 2$, let $v(P, \{x\}) \equiv x$ for all $x \in X$.

[23]Apesteguia et al. (2014) call this condition *Condorcet loser consistency* (a name that usually refers to the requirement that the Condorcet loser cannot be chosen). To avoid potential confusion with this weaker property, I depart from their nomenclature.

[24]While IIA was first proposed for individual choice, similar conditions have also been considered for voting (Fishburn 1974, Young 1995, Ching 1996). To be clear, the condition is distinct from Arrow's (1950) IIA condition for social choice (Ray 1973).

[25]Nonetheless, ILA is independent from IS. To see this, consider the decision rule v on $X := \{x_1, x_2, x_3\}$ that selects: the majority preferred alternative between x_1 and x_2 when both are available, and selects x_i on $\{x_i, x_3\}$. Since $(\{x_1, x_3\}, \{x_2, x_3\})$ splits $\{x_1, x_2, x_3\}$, v satisfies IS. To see that it violates ILA, consider a profile P where x_3 is the Condorcet winner on X. If v satisfies ILA, then $v(P, \{x_1, x_2, x_3\}) = \cdots = x_3$. But this contradicts the assumption that $v(P, \{x_1, x_2, x_3\}) = v(P, \{x_1, x_2\}) \neq x_3$.

unique splitting. To establish sufficiency, it remains to show only that the agenda $\mathbb{S}^v$ is simple and implements v. The uniqueness of the implementing agenda follows, in turn, from the uniqueness of the splittings.

To get a better sense of the role played by IS, consider what happens when this condition is strengthened to Plott's path independence. For decision rules, Plott's condition implies that, for each profile P, the associated choice function $v(P, \cdot)$ can be rationalized by a linear ordering. In turn, ILA implies that majority rule determines the outcome on binary issues. In combination, these observations imply that for each profile P, the majority relation must be a linear ordering. While true for certain profiles, this is clearly not true for all profiles. To avoid this problem while preserving the connection to majority rule, the scope of Plott's condition must be limited. Instead of being unaffected by *every* splitting of an issue, the outcome is unaffected only by *one* such splitting (again see Claim 5 of the Appendix).

4. Marginalization on specialized formats

Rules for determining the order of voting on proposals have a significant impact on agenda structure. In this section, I define two classes of simple agendas, called *priority agendas* and *convex agendas*, whose structure is closely related to the most prevalent order-of-voting rules used in legislative settings. I then show that each class is characterized by the way that it marginalizes (or discriminates against) certain alternatives.

4.1 *Priority agendas*

Priority agendas share the same basic sequential structure as Euro–Latin and Anglo–American agendas: among the remaining alternatives, each question contests the two alternatives that have the highest priority.

Definition 6. A *priority agenda* on $A \in \mathbf{X}$ is a simple agenda, equipped with a linear ordering $\succ$ on A, such that, for every nonterminal node D of the agenda, $x \succ y \succ z$ and $x, y \in D$ imply $z \in D$. In the sequel, I refer to an ordering that satisfies this condition as a *priority ordering* for the agenda.

Without loss of generality, the priority $\succ$ may be defined so that every question eliminates either the remaining alternative that has the highest priority or an interval of alternatives starting from the remaining alternative that has the second highest priority (see Remark 6 of the Appendix). Given a node D, label the alternatives $d_1, \ldots, d_k \in D$ according to $\succ$ (so that $d_i \succ d_j$ implies $i < j$ for all $d_i, d_j \in D$). Then, for some index i^* such that $3 \le i^* \le k+1$, the agenda must subdivide the issue D as in Figure 3.[26,27]

[26] Simplicity imposes restrictions on the next highest priority alternative d_{i^*} that is reachable when the majority support d_1. An earlier version of the paper defined priority agendas in an equivalent (but cumbersome) way that makes these restrictions explicit.

[27] In case $i^* = k+1$, the successors of $D = \{d_1, \ldots, d_k\}$ are (the singleton) d_1 and the complementary issue $\{d_2, \ldots, d_k\}$.

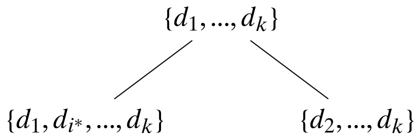


FIGURE 3. Structure of a priority agenda at a nonterminal node D.

Among priority agendas, Euro–Latin and Anglo–American agendas define the two extremes. At each node of a Euro–Latin agenda, majority support for the highest priority alternative d_1 forecloses every other alternative reachable from that node (i.e., $i^* = k + 1$). For an Anglo–American agenda, majority support for the highest priority alternative d_1 forecloses only the second highest priority alternative d_2 (i.e., $i^* = 3$).

Priority agendas are closely related to the rule of precedence that many Anglo–American (and some European) legislatures use for voting on amendments.[28,29] This rule states that a new proposal must be put to a vote before (i) the proposal that it amends and (ii) any subsequent amendment of the same proposal.

This rule typically induces a priority among the amended versions of the original proposal. In the textbook scenario with a proposal, an amendment to the proposal, an amendment to the amendment, etc., the rule of precedence ensures that "more" amended versions of the proposal have priority over "less" amended versions. Depending on whether the more amended versions must then be put to a vote against less amended versions, the agenda induced by this priority could be Euro–Latin or Anglo–American (Miller 1995, pp. 13 and 18).

Example 1 illustrates another situation where the rule of precedence induces a priority among versions of the original proposal. This agenda gives highest priority to the amended bill ba', intermediate priority to the amended bill ba'' and the original bill b, and lowest priority to the status quo alternative $\varnothing$. However, a slight variation of this example shows that the rule of precedence does not necessarily induce a priority.

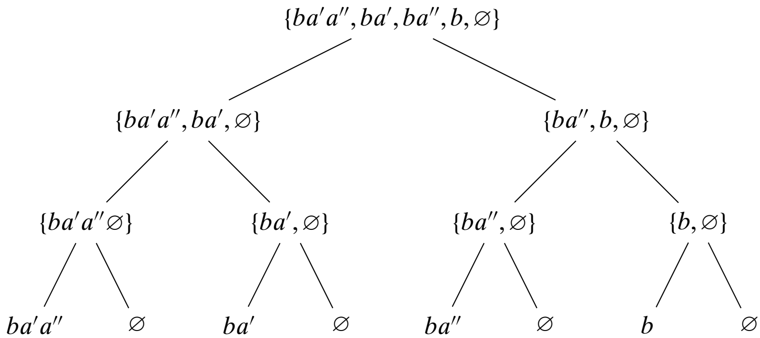
EXAMPLE 5. When the two amendments a' and a'' in Example 1 are compatible, a small change to the agenda occurs: a'' must be recognized regardless of whether a' is adopted. This results in the following agenda.[30]

[28] As noted by Bach (1983, p. 575), this rule is used by both houses of the U.S. Congress.

[29] While typically associated with the Anglo–American tradition, this rule is also used by some countries with a Euro–Latin legal tradition, including the Czech Republic (Zákon o jednacḿ řádu Poslanecké sněmovny, Article 72), Portugal (Regimento da Assembleia da República, Article 154), and Slovakia (Rokovací poriadok Národnej Slovenskej republiky, Article 37).

[30] See Figure 1.2 of Sullivan (1984), Figure 3 of Ordeshook and Schwartz (1987), Figure 3 of Miller (1995), or Figure 2 of Schwartz (2008).

Q1. *Shall the first amendment a' to the bill b be adopted?*
Q2. *Shall the second amendment a'' to the bill b be adopted?*
Q3. *Shall the perfected bill replace the status quo $\varnothing$?*



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Relative to Example 1, the only change is to allow for the "doubly" amended bill $ba'a''$. Somewhat surprisingly, the resulting agenda is not compatible with a priority. To see this, suppose that b had highest priority. Then the other versions of the bill must all appear in the *other* subtree below the root (as in Figure 3), which is not the case. Since the same argument holds for any version of the bill, the agenda is not prioritarian. Still, it is simple. Since every node has a unique label, it is history-independent. To see that it is also persistent, note that the status quo $\varnothing$ is the only uncontested alternative and it is always contested by the final question.

4.2 *Convex agendas*

Just like priority agendas, convex agendas are defined in terms of a linear ordering of the alternatives. The difference is that each question contests the two remaining alternatives that are furthest apart (or most "extreme") in terms of this ordering rather than the two that are most highly ranked.

Definition 7. A *convex agenda* on $A \in \mathbf{X}$ is a simple[31] agenda, equipped with a linear ordering $\succ$ on A, such that, for every nonterminal node D of the agenda, $x \succ y \succ z$ and $x, z \in D$ imply $y \in D$. In the sequel, I refer to an ordering that satisfies this condition as a *convex ordering* for the agenda.

This definition implies that every question eliminates an interval of alternatives starting from one of the two extremes. For a given node $D = \{d_1, \dots, d_k\}$ (with the alternatives labeled according to $\succ$), this means that the agenda must subdivide the issue D as in Figure 4 for some indices $\hat{\imath}, \hat{\jmath}$ such that $1 < \hat{\jmath} \leq \hat{\imath} < k$.

[31]The definition of Kleiner and Moldovanu (2017) is more general since it does not require the agenda to be simple.

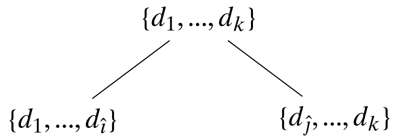


FIGURE 4. Structure of a convex agenda at a nonterminal node D.

Clearly, convexity rules out certain agenda formats. Anglo–American agendas, for instance, are convex only if they contain three or fewer alternatives.[32] In contrast, all Euro–Latin agendas are convex: at any node, majority support for d_1 forecloses every other alternative, while majority support against d_1 forecloses only d_1 itself. The convexity of Euro–Latin agendas follows from their knockout structure. Indeed, to define a convex ordering for *any* knockout agenda, it suffices to "read off" the terminal nodes from left to right.

A range of convex agendas besides knockout agendas arise in legislative settings. The reason is their close relationship to the *extremeness rule* that most European legislatures use for voting on proposals.[33] Unlike the rule of precedence, this rule is primarily content-based. It orders proposals in terms of the "extremeness" of their departure from an established reference point (typically the original proposal or the status quo) and puts more extreme proposals to vote before less extreme ones.[34]

Since it imposes no restrictions on (the structure of) the convex ordering, Definition 7 is flexible enough to capture the various notions of extremeness used in different jurisdictions. In fact, it even captures agendas, like the agenda in Example 3, that are built by contesting the *least* extreme alternatives. (For this agenda, one convex ordering of the amendments is $a_3 \succ a_4 \succ a_1 \succ a_2$.)

It is not difficult to show that a simple agenda with fewer than five alternatives must either be prioritarian or convex.[35] With more alternatives, a simple agenda need not respect either of these formats. Example 5 serves to illustrate. As explained, the agenda in this example is simple but not prioritarian. To see that it is not convex, simply note

[32] To establish this point (which does not appear to be clearly appreciated in the literature), fix an Anglo–American agenda $\mathtt{T}_A$ such that $|A| \geq 4$ and index $A \equiv \{a_1, \ldots, a_k\}$ by an ordering that makes it prioritarian. By way of contradiction, suppose that there is another ordering $\succ_e$ that makes $\mathtt{T}_A$ convex. Then, for the two nodes below the root (labeled $\{a_1, a_3, \ldots, a_k\}$ and $\{a_2, a_3, \ldots, a_k\}$), Figure 4 implies $a_1 \succ_e a_4 \succ_e a_3$ and $a_3 \succ_e a_4 \succ_e a_2$.

[33] Besides countries with an Anglo–American legislative tradition (United Kingdom, Switzerland, and Sweden), some notable exceptions are the three countries (Czech Republic, Portugal, and Slovakia) mentioned in footnote 29 (Rasch 2000, Table 2).

[34] See, e.g., Article 30.2 of the German Geschäftsornung des Bundesrates: "...if several proposals are made to the same subject, then the first vote shall be on the farthest-reaching proposal. Decisive is the degree of deviation from status quo."

[35] This is clear for agendas with three or fewer alternatives. It is also true for agendas with four alternatives. Up to permutation, there are nine simple agendas with four alternatives: four that are convex and prioritarian (the Euro–Latin agenda plus three others); two priority agendas that are not convex (the Anglo–American agenda and the one in Example 1); three convex agendas that are not prioritarian (the agendas in Examples 2 and 3 plus one other).

that any convex ordering $\succ$ of the alternatives must put the status quo $\varnothing$ between *every* pair of perfected bills in $\{ba'a'', ba', ba'', b\}$. This is a contradiction.

Remark 2. Every simple agenda with four or fewer alternatives is either prioritarian or convex. However, there are simple agendas with more than four alternatives that do not respect either format.

4.3 *Characterization*

For Euro–Latin and Anglo–American agendas, Moulin (1988, p. 250) showed that the lowest priority alternative is chosen only if it clearly appeals to the majority. Formally, an alternative $a \in A$ is the *Condorcet winner* for the decision problem $(P, A) \in \mathbf{P} \times \mathbf{X}$ if $aM_P x$ for all $x \in A \setminus a$. A decision rule v *marginalizes* $a^* \in A$ on the issue $A \in \mathbf{X}$ if, for all profiles $P \in \mathbf{P}$, $v(P, A) = a^*$ only if a^* is the Condorcet winner for (P, A).

In fact, Euro–Latin and Anglo–American agendas can be distinguished from other simple agendas by the *number* of alternatives that they marginalize. For every issue consisting of three or more alternatives, Anglo–American agendas marginalize one alternative, whereas Euro–Latin agendas marginalize two alternatives. (Intuitively, the difference comes down to the fact that the two alternatives with lowest priority are symmetrically placed in a Euro–Latin agenda but not in an Anglo–American agenda.)

Theorem 2. *(a) A decision rule v is implementable by Euro–Latin agenda if and only if it satisfies IS and ILA, and, for every issue $A \in \mathbf{X}$ consisting of three or more alternatives, v marginalizes two alternatives.*

(b) A decision rule v is implementable by Anglo–American agenda if and only if it satisfies IS and ILA, and, for every issue $A \in \mathbf{X}$ consisting of three or more alternatives, v marginalizes one alternative.

Despite the important structural differences between Euro–Latin and Anglo–American agendas, Theorem 2 shows that the two formats marginalize alternatives in a similar fashion. This result provides a counterpoint to the work of Apesteguia et al. (2014). Instead of emphasizing fundamental similarities between the two formats, their characterizations (which I discuss in the Online Appendix) highlight some key differences.

More generally, marginalization may be used to characterize the voting outcomes associated with all priority agendas. Like a Euro–Latin or an Anglo–American agenda, a priority agenda always marginalizes the lowest priority alternative. Accordingly, a decision rule implemented by such an agenda must satisfy the following condition.

Mandatory marginalization. *For every issue $A \in \mathbf{X}$, there is some alternative that v marginalizes on A.*

In fact, this condition distinguishes priority agendas from all other simple agendas.

Theorem 3. *A decision rule v is implementable by priority agenda if and only if it satisfies IS, ILA, and mandatory marginalization.*

Convex agendas can be characterized along similar lines. As Example 2 serves to illustrate, a convex agenda need not marginalize any alternatives. In fact, for issues consisting of four or more alternatives, a convex agenda marginalizes one alternative only in the interest of promoting another. To formalize, fix an issue $A \in \mathbf{X}$, where the decision rule v marginalizes alternative $a_* \in A$. Then *v marginalizes a_* in favor of $a^* \in A \setminus a_*$ on A* if $v(P, A) = a^*$ for some profile $P \in \mathbf{P}$ such that $a_* M_P a^*$.

Directed marginalization. *For every issue $A \in \mathbf{X}$ that consists of four or more alternatives, each alternative $a_* \in A$ that v marginalizes on A is in favor of some other alternative $a^* \in A$.*

This condition distinguishes convex agendas from all other simple agendas.

Theorem 4. *A decision rule v is implementable by convex agenda if and only if it satisfies IS, ILA, and directed marginalization.*

5. Monotonicity of simple agendas

May (1952) argued that a chosen alternative should continue to be chosen when it improves in terms of voters' preferences.[36] In this section, I show that all decision rules implemented by simple agenda satisfy this desirable monotonicity condition, while decision rules implemented by agendas outside this class need not.

To formalize, let $P^{\uparrow x}$ denote a profile where every voter's preference is identical to profile P except for one voter, whose preference between x and the immediately preferred alternative (if any) is reversed. Thus, $P^{\uparrow x}$ differs from P only by improving alternative x for a single voter. Then a decision rule v is (preference) *monotonic* at the issue A if, for each profile $P \in \mathbf{P}$ and every profile $P^{\uparrow x} \in \mathbf{P}$, $v(P, A) = x$ implies $v(P^{\uparrow x}, A) = x$.

Preference monotonicity. *For every issue $A \in \mathbf{X}$, v is monotonic.*

This condition translates May's positive responsiveness to the context of single-valued decision rules.[37]

According to a folk result, every decision rule implemented by knockout agenda must be monotonic (see, e.g., Moulin 1988, p. 285 or Altman et al. 2009). To see this, consider the "backward induction" path that leads to the outcome x being chosen for the problem (P, X). Since each alternative appears only once in the agenda, the alternatives that x faces along this path cannot be affected by improving it. As a result, x remains the outcome for any problem $(P^{\uparrow x}, X)$. Moulin (1986, p. 288) showed that this

[36]The normative appeal of this property should be clear. As a weak form of Maskin monotonicity, it also has practical appeal. In particular, it discourages certain kinds of voter misrepresentations Sanver and Zwicker (2009) as well as certain kinds of manipulations by the alternatives, which may represent candidates in an election (Altman et al. 2009).

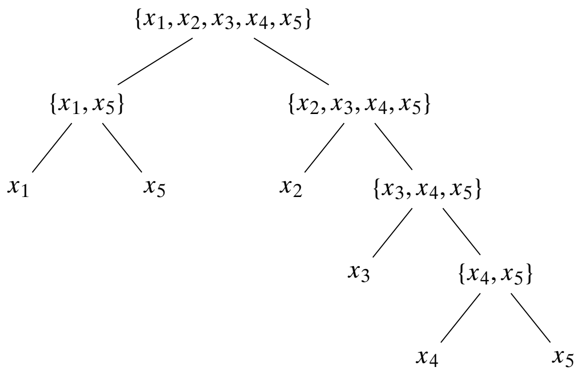
[37]May originally defined positive responsiveness for social aggregation rules rather than social choice rules. When translated to the setting of (multivalued) choice, May's condition is usually understood to be stronger than preference monotonicity. Conventionally, a (multivalued) choice rule $v : \mathbf{P} \times \mathbf{X} \to \mathbf{X}$ is said to be positively responsive at $A \in \mathbf{X}$ if $x \in v(P, A)$ implies $v(P^{\uparrow x}, A) = \{x\}$ and to be monotonic at $A \in \mathbf{X}$ if $x \in v(P, A)$ implies only $x \in v(P^{\uparrow x}, A)$. See Horan et al. (2019) for a discussion.

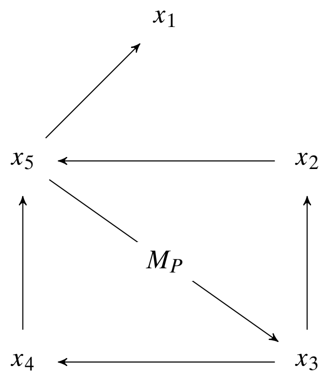
type of argument extends to Anglo–American agendas. Even though some alternatives appear more than once, improving the winner x cannot affect which alternatives it faces along a winning path. In fact, the same reasoning extends to *all* simple agendas.

Theorem 5. *Every decision rule v implementable by simple agenda satisfies preference monotonicity.*

Beyond the class of simple agendas, it is not difficult to identify non-monotonic decision rules.

Example 6. Consider the nonpersistent agenda on $X = \{x_1, x_2, x_3, x_4, x_5\}$ depicted below *(left)*.





The decision rule implemented by this agenda is non-monotonic. To see this, consider a profile P whose majority relation completes the partial relation M_P depicted above *(right)*. The sophisticated outcome at P is x_5. However, when x_5 improves by reversing the majority comparison with x_4, the outcome changes to x_2. ♢

What makes the agenda in this example troubling is that it has two features common in legislative settings. It is *nonrepetitive*: every question eliminates some potential outcomes.[38,39] It is also *continuous*: for every question, some contested outcome continues to be contested until it is eliminated or eventually selected.[40] Note that every simple agenda has these same two features (by Remark 7 of the Online Appendix).

The next remark follows by extending the agenda in Example 6 to more alternatives.

[38]An agenda is nonrepetitive if, for every nonterminal node q, $\ell(q) \neq \ell(q_i)$ for $i = y, n$. This is Farquharson's (1969) Axiom II.

[39]The non-monotonic agenda proposed by Moulin (1986, p. 284) lacks this feature.

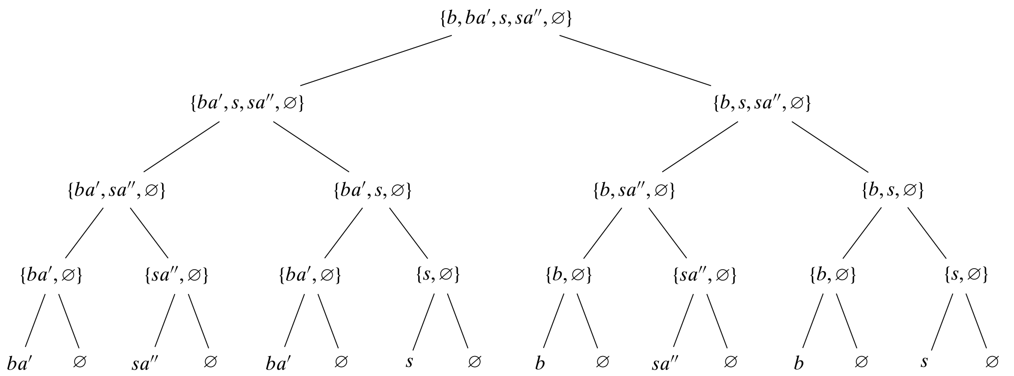
[40]An agenda is continuous if, for every nonterminal node q such that $c(q_i) \neq \varnothing$ (for $i = y$ or n), some alternative in $c(q_i)$ labels exactly one terminal node below q_i. Ordeshook and Schwartz (1987, p. 185) define this concept only for "Ordeshook–Schwartz agendas" (see footnote 9). To adapt it to "Farquharson–Miller agendas," Miller (1995, p. 26) requires *every* contested alternative to be contested until it is eliminated or selected. The weaker definition given here is closer to Groseclose and Krehbiel (1993).

REMARK 3. There are decision rules implemented by nonrepetitive and continuous agendas that violate preference monotonicity if (and only if) the universe of alternatives X contains five or more alternatives.[41]

This remark shows that nonrepetitiveness and continuity are not sufficient for preference monotonicity. The next example, which features an agenda sometimes used by the U.S. Congress, shows that the second feature is also unnecessary.

EXAMPLE 7. Both the House and the Senate have special procedures for parallel consideration of a bill and a substitute to the bill. When amendments to the original bill and substitute are proposed, the rules specify a "two-stage" agenda where the amendments are put to a vote before deciding on the form of the bill.[42]

Q1. *Shall the amendment a' to the original bill b be adopted?*
Q2. *Shall the amendment a'' to the substitute bill s be adopted?*
Q3. *Shall the bill (as amended) be adopted over the substitute (as amended)?*
Q4. *Shall the perfected version of the bill be selected over the status quo $\varnothing$?*



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Since this agenda is history-independent, it is nonrepetitive (by Claim 9 of the Appendix). At the same time, it is also discontinuous: the second question shifts the focus from the amendment a' of the original bill to the amendment a'' of the substitute bill.[43] Nonetheless, it implements a monotonic rule.

REMARK 4. The agenda in Example 7 implements a decision rule that satisfies preference monotonicity.

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[41] Every nonrepetitive and continuous agenda on $|X| \leq 4$ alternatives implements a preference monotonic decision rule. As such, the counterexample in Example 6 is minimal in terms of the number of alternatives (as well as the number of terminal nodes).

[42] See also Figure 2 of Ordeshook and Schwartz (1987), Figure 2 of Banks (1989), Figure 4 of Miller (1995), or Figure 2 of Schwartz (2008).

[43] Formally, b and ba' are contested at the root, but s and sa'' are contested at the two successors of the root. This shows that the agenda is also nonpersistent: the set of uncontested alternatives at the root $\{s, sa''\}$ does not label any node below the root.

The proof leverages the fact that both sub-agendas below the root (i.e., starting from $\{ba', s, sa'', \varnothing\}$ and $\{b, s, sa'', \varnothing\}$) are simple. Yet, as Example 6 illustrates, not every agenda formed by joining two simple agendas at the root implements a monotonic decision rule. Indeed, as Example 7 suggests, the relative position of the alternatives that appear in both sub-agendas seems to play a crucial role.

6. Conclusion

In the current paper, I introduce a broad class of agendas, called simple agendas, whose two defining features are common in legislative settings. My main result (Theorem 1) characterizes the rules implemented by simple agenda. I also show that all simple agendas exhibit a desirable monotonicity property (Theorem 5) originally proposed by May (1952). Finally, I characterize some specialized agenda formats in terms of the way that they deviate from neutrality by marginalizing certain alternatives (Theorems 2–4).

These results establish the connection between form and function for a wide range of agendas used in legislative settings. Ultimately, the goal is to provide insights that can be leveraged in applications. In a companion work, I consider implications of my results for applications related to strategic candidacy (Dutta et al. 2001, 2002) and competitive agenda-setting (Dutta et al. 2004, Duggan 2006).

I close by mentioning a few questions raised by the results of the current paper. Given Theorems 2–4, one natural question is whether marginalization is equally important for other simple agenda formats that arise in legislative settings. In the Online Appendix, I discuss some progress on this question. Another natural question relates to monotonicity. More specifically, Theorem 5 and Remark 4 suggest that agendas used in legislative settings tend to implement preference monotonic decision rules. Ideally, one would hope to identify basic structural features of legislative agendas (even more fundamental than history-independence and persistence) that are sufficient to ensure monotonicity.

Appendix

Note. For ease of presentation, I frequently abuse notation by referring to a node q of an agenda by its label $\ell(q)$. (Since I do so only when the agenda is history-independent, this creates no possibility for confusion.) I also follow the convention that $\{v(P, \varnothing)\} \equiv \varnothing$ (instead of $\{v(P, \varnothing)\} = \{\varnothing\}$).

A.1 *Primer on graph theory*

A *graph* $\mathtt{G} = (Q, E)$ is a pair consisting of a collection of nodes Q and a collection of edges E, each of which connects two nodes in Q. It is a *labeled* graph if each node $q \in Q$ has a label $\ell(q)$, *finite* if $|Q|$ is finite, and *loopless* if there exists no edge in E (called a *loop*) that connects a node in Q to itself. A *simple path* in $\mathtt{G}$ is a nonrepeating sequence of nodes in Q such that each adjacent pair in the sequence is connected by an edge in E.

A *tree* $\mathtt{T} = (Q, E)$ is a loopless graph such that every pair of distinct nodes in Q is connected by a unique simple path. In a tree, nodes are ordered in a natural way in terms of betweenness. In particular, a node $q' \in Q$ is *between* two nodes $q, q'' \in Q$ if the path from q to q'' (inclusive of the end nodes) passes through q'.

A tree is *rooted* if one node $r \in Q$ is specially designated as the root. In a rooted tree $\mathtt{T}_r \equiv (Q, E, r)$, betweenness defines a partial ordering $\leq_E$ on Q, where $q \leq_E \tilde{q}$ for $q, \tilde{q} \in Q$ if q is between the root r and the node $\tilde{q}$. (It is clear that $\leq_E$ is reflexive, transitive, and antisymmetric.) Then $\tilde{q} \in Q$ is a *predecessor* (resp. *successor*) of $q \in Q$ if $\{q, \tilde{q}\} \in E$ and $\tilde{q} \leq_E q$ (resp. $q \leq_E \tilde{q}$). A node $q \in Q$ is *terminal* if it has no successors and is *nonterminal* otherwise. By construction, every nonroot node has a unique predecessor and every nonterminal node has a successor.

Finally, a *binary tree* is a rooted and labeled tree such that every nonterminal node in Q has exactly two successors. In the main text, I sometimes refer to the two successors of a nonterminal node as *siblings*.

A.2 *Proof of Remark 1*

Suppose that v is implemented by $\mathtt{T}$. Then $v(P^A, X) = \text{SVO}[\mathtt{T}; P^A]$ and $v(P, A) = \text{SVO}[\mathtt{T}|_A; P]$. To see that $\text{SVO}[\mathtt{T}; P^A] = \text{SVO}[\mathtt{T}|_A; P^A]$, note that the solution concept selects the Condorcet winner in any terminal subgame. Then "backward induction" allows one to delete the Condorcet loser and repeat the argument on the resulting agenda. Since the profiles P^A and P coincide on A, $\text{SVO}[\mathtt{T}|_A; P^A] = \text{SVO}[\mathtt{T}|_A; P]$ as well. Combining all of these equalities gives $v(P^A, X) = \text{SVO}[\mathtt{T}; P^A] = \text{SVO}[\mathtt{T}|_A; P^A] = \text{SVO}[\mathtt{T}|_A; P] = v(P, A)$.

A.3 *Remark about persistence*

Remark 5. For an agenda $\mathtt{T}$, the following statements are equivalent: (a) $\mathtt{T}$ is persistent; (b) for each node q of $\mathtt{T}$ such that $u(q) \neq \varnothing$ and every node $\tilde{q}$ below q such that $\ell(\tilde{q}) \supset u(q)$, either (i) $\{\ell(\tilde{q}_y), \ell(\tilde{q}_n)\} = \{\ell', u(q)\}$ or (ii) $\{\ell(\tilde{q}_y), \ell(\tilde{q}_n)\} = \{\ell', \ell''\}$ for $\ell', \ell'' \supset u(q)$.

Note. For ease of presentation in the sequel, I refer to the property specified in (b) above as Property $(*)$.

Proof. (a) $\Rightarrow$ (b). Fix a node $\tilde{q}$ below q such that $\ell(\tilde{q}) \supset u(q) \neq \varnothing$. By persistence, $u(q) \subseteq \ell(\tilde{q}_y)$ or $\ell(\tilde{q}_n)$. Otherwise, there is no node between q and a terminal node t below $\tilde{q}$ with a successor labeled by $u(q)$. If $u(q) = \ell(\tilde{q}_y)$ or $\ell(\tilde{q}_n)$, then case (i) of Property $(*)$ obtains. Otherwise, $\ell(\tilde{q}_y), \ell(\tilde{q}_n) \supset u(q)$ so that case (ii) obtains.

(b) $\Rightarrow$ (a). The proof is by strong induction on $|X| \equiv m$. The base cases $m = 2, 3$ are trivial. To complete the induction, suppose that the claim holds for $m \leq n$ and consider $m = n + 1$. Let q_y^1 and q_n^1 denote the successors of the root r in $\mathtt{T}$. Note that the subtrees $\mathtt{T}(q_y^1)$ and $\mathtt{T}(q_n^1)$ inherit Property $(*)$ from $\mathtt{T}$. As such, $\mathtt{T}(q_y^1)$ and $\mathtt{T}(q_n^1)$ are persistent by the induction hypothesis. Consequently, $\mathtt{T}$ is persistent at every nonterminal node, with the possible exception of r.

To check persistence at r, suppose that $u(r) \neq \varnothing$. Consider a terminal node t below q_y^1 and (without loss of generality) let $q_y^1, \ldots, q_y^k = t$ denote the path from q_y^1 to t. By Property

$(*)$, there are two possibilities for the successors of q_y^1: (i) $\{\ell(q_y^2), \ell(q_n^2)\} = \{\ell', u(r)\}$; (ii) $\{\ell(q_y^2), \ell(q_n^2)\} = \{\ell', \ell''\}$ for $u(r) \subset \ell', \ell''$. In case (i), q_y^1 is a node between r and t with a successor labeled $u(r)$. In case (ii), consider the node q_y^2. Since $u(r) \subset \ell(q_y^2) \subset \ell(q_y^1)$, one can repeat the same argument for q_y^2. Since $\mathtt{T}$ is finite, it follows that one of the nodes q_y^i between r and t must have a successor labeled $u(r)$. Since the same argument holds for any terminal node t' below q_n^1, $\mathtt{T}$ is persistent at r. □

A.4 *Proof of Theorem 1*

A.4.1 *Sufficiency*

Claim 1. *For $A = B \cup C$, the following statements are equivalent: (a) $B, C \neq A$, (b) $B \cap C \neq B, C$, and (c) $B \setminus C, C \setminus B \neq \varnothing$.*

Proof. $(a) \Rightarrow (b)$. Suppose that $B \cap C = B$. Then $C = (B \cap C) \cup C = B \cup C = A$, which is a contradiction.

$(b) \Rightarrow (c)$. Suppose that $B \setminus C = \varnothing$. Then $B = (B \cap C) \cup (B \setminus C) = (B \cap C) \cup \varnothing = B \cap C$, which is a contradiction.

$(c) \Rightarrow (a)$. Suppose that $B = A$. Then $C \setminus B = C \setminus A = \varnothing$, which is a contradiction. □

Note. For each of Claims 2–8, I assume that v satisfies IS and ILA.

Claim 2. *If $P, P' \in \mathbf{P}$ coincide on $A \in \mathbf{X}$, then $v(P, A) = v(P', A)$.*

Proof. The proof is by induction on $|A|$. The case $|A| = 2$ follows from ILA. Suppose that the claim holds for $|A| = n$ and consider $|A| = n + 1$. Let (B, C) denote the splitting of A. Then IS, the induction hypothesis, and the base case imply $v(P, A) = v(P, \{v(P, B), v(P, C)\}) = v(P, \{v(P', B), v(P', C)\}) = v(P', \{v(P', B), v(P', C)\}) = v(P', A)$. □

Claim 3. *If $A \in \mathbf{X}$ and P^A is as defined in Remark 1, then $v(P, A) = v(P^A, X)$.*

Proof. By ILA, $v(P^A, X) = \cdots = v(P^A, A)$. Since $v(P^A, A) = v(P, A)$ by Claim 2, $v(P^A, X) = v(P, A)$. □

Claim 4. *If (B, C) splits $A \in \mathbf{X}$ and $D \subset A$, then (i) $v(P, D) = v(P, \{v(P, B \cap D), v(P, C \cap D)\})$ and (ii) $(B \cap D, C \cap D)$ splits D if $D \neq B \cap D, C \cap D$.*

Proof. Fix $x \in A$ and let (B, C) denote the splitting of A. Let $P^{X \setminus x}$ coincide with P except that x is demoted to the bottom of each voter's preference. Then $v(P, A \setminus x) = v(P^{X \setminus x}, A \setminus x) = v(P^{X \setminus x}, A) = v(P^{X \setminus x}, \{v(P^{X \setminus x}, B), v(P^{X \setminus x}, C)\}) = v(P, \{v(P, B \setminus x), v(P, C \setminus x)\})$ by Claim 2, ILA, and IS. Part (i) follows by repetition of this reasoning. For part (ii), observe that $D \neq B \cap D, C \cap D$ implies $B \cap C \cap D \neq B \cap D, C \cap D$ by Claim 1. By part (i), it follows that $(B \cap D, C \cap D)$ splits D. □

Claim 5. *Every issue $A \in \mathbf{X}$ has a unique v-splitting.*

Proof. The proof is by induction on $|A|$. The case $|A| = 2$ is obvious. For the case $|A| = 3$, Table 1 shows that every exact bi-cover of A yields a distinct pair of outcomes on the two Condorcet triples. It follows that the splitting must be unique.

Now suppose that the claim holds for $|A| = n$ and consider $|A| = n + 1$. By way of contradiction, suppose that (B, C) and (B', C') are distinct splittings of A. If both partition A, then there is some $x \in A$ such that $(B \setminus x, C \setminus x)$ and $(B' \setminus x, C' \setminus x)$ are distinct partitions of $A \setminus x$. Then, by Claim 4(ii), this contradicts the induction hypothesis. Suppose that $x \in B \cap C$ for some $x \in A$. If $x \in B' \cap C'$, then $(B \setminus x, C \setminus x)$ and $(B' \setminus x, C' \setminus x)$ are distinct splittings of $A \setminus x$ by Claim 4(ii), which again contradicts the induction hypothesis. Finally, suppose that $x \in B' \setminus C'$. There are two possibilities for $(B \setminus x, C \setminus x)$ and $(B' \setminus x, C' \setminus x)$: either (i) they coincide or (ii) they are distinct.

(i) Suppose that $B \setminus x = B' \setminus x$ and $C \setminus x = C'$. (The reasoning is symmetric for $B \setminus x = C'$ and $C \setminus x = B' \setminus x$.) Since (B, C) splits A, there exist $b \in B \setminus C$ and $c \in C \setminus B$. Since $B \setminus x = B' \setminus x$ and $C \setminus x = C'$, $b \in B' \setminus C'$ and $c \in C' \setminus B'$. Letting $D = \{b, c, x\}$ gives distinct splittings $(B \cap D, C \cap D) = (\{x, b\}, \{x, c\})$ and $(B' \cap D, C' \cap D) = (\{x, b\}, \{c\})$ by Claim 4(ii), which contradicts the base case of the induction.

(ii) First, suppose that $B' \setminus C' \neq x$. Then $(B \setminus x, C \setminus x)$ and $(B' \setminus x, C' \setminus x)$ are distinct splittings of $A \setminus x$ by Claim 4(ii), which contradicts the induction hypothesis. Next suppose that $B' \setminus C' = x$. Since (B, C) splits A, there exist alternatives $b \in B \setminus C$ and $c \in C \setminus B$. Letting $D = \{b, c, x\}$ gives $(B \cap D, C \cap D) = (\{x, b\}, \{x, c\})$, $x \in B' \setminus C'$, and $b, c \in C'$, so (a) $(B' \cap D, C' \cap D) = (\{x\}, \{b, c\})$, (b) $(B' \cap D, C' \cap D) = (\{x, b\}, \{b, c\})$, or (c) $(B' \cap D, C' \cap D) = (\{x, b, c\}, \{b, c\})$. Subcases (a) and (b) yield splitting of D that differs from $(B \cap D, C \cap D)$, which contradicts the base case of the induction by Claim 4(ii). Finally, case (c) yields a contradiction. By Claim 4(i),

$$\begin{aligned} v(P_{bcx}, D) &= v\big(P_{bcx}, \{v\big(P_{bcx}, B' \cap D\big), v\big(P_{bcx}, C' \cap D\big)\}\big) \\ &= v\big(P_{bcx}, \{v(P_{bcx}, D), v\big(P_{bcx}, \{b, c\}\big)\}\big). \end{aligned}$$

The solution is $v(P_{bcx}, D) = b$, but $v(P_{bcx}, D) = v(P_{bcx}, \{v(P_{bcx}, B \cap D), v(P_{bcx}, C \cap D)\}) = c$ as well. ☐

Claim 5 makes it possible to define a simple agenda for a decision rule v. Let $\mathrm{S}_{\{x,y\}}$ denote the unique simple agenda with two terminal nodes labeled x and y, and let $\mathrm{S}_{\{x\}}$ denote the unique degenerate agenda with a single node labeled x.

Definition 8. Given a decision rule v that satisfies IS and ILA, the agenda S^v is defined as follows:

(i) First, define $\mathrm{S}^v(1) \equiv \mathrm{S}_{\{X\}}$ (to be the degenerate agenda whose only node is labeled X).

(ii) Then, for each $j > 1$, define the agenda $\mathrm{S}^v(j + 1)$ from $\mathrm{S}^v(j)$ as follows:

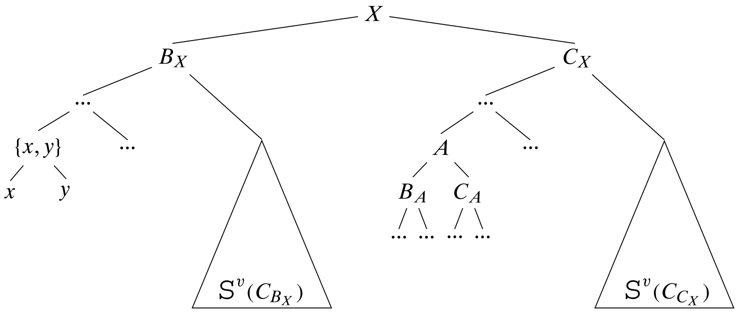


FIGURE 5. The recursive construction of S^v.

- For each terminal node labeled by a nonsingleton A (if any), determine its v-splitting (B_A, C_A).
- Replace every terminal node of $S^v(j)$ labeled by A with the simple agenda $S_{\{B_A,C_A\}}$.

(iii) Finally, define $S^v \equiv S^v(|X|-1)$.

Figure 5 illustrates the recursive construction of S^v. The leftmost nodes below B_X illustrate the construction for $|A|=2$, while the leftmost nodes below C_X illustrate it for $|A|>2$. In turn, the two triangles represent the subgames starting from the nodes labeled C_{B_X} and C_{C_X}, while the ellipses indicate where details have been omitted.[44]

By construction, the agenda S^v is history-independent. The next three claims show that it is also persistent:

CLAIM 6. *If (B,C) splits $A \in \mathbf{X}$ and (B',C') splits $D \in \mathbf{X}$ for $\varnothing \subset B \cap C \subset D \subseteq C$, then $B \cap C \subseteq B'$ or $B \cap C \subseteq C'$.*

PROOF. By way of contradiction, suppose that $B \cap C \nsubseteq B', C'$. Fix alternatives $b \in (B \cap C) \setminus C'$ and $c \in (B \cap C) \setminus B'$. Since $B \cap C \subset D$, there exists some $d \in D \setminus (B \cap C)$. Up to symmetry, there are two cases: (i) $d \in B' \cap C'$ and (ii) $d \in B' \setminus C'$. Since (B,C) splits A, $v(P,\{b,c,d\}) = v(P,\{v(P,\{b,c\}), v(P,\{b,c,d\})\})$ by Claim 4(i). I show that both cases (i) and (ii) entail a contradiction.

Case (i). Since (B',C') splits D, Claim 4(i) implies

$$
\begin{aligned}
v\big(P,\{b,c,d\}\big) &= v(P,\big\{v\big(P,B' \cap \{b,c,d\}\big), v\big(P,C' \cap \{b,c,d\}\big\}\big) \\
&= v\big(P,\big\{v\big(P,\{b,d\}\big), v\big(P,\{c,d\}\big)\big\}\big).
\end{aligned}
$$

Consequently, $v(P_{bcd},\{b,c,d\}) = c$ for the Condorcet triple P_{bcd}. Combining this with the formula in the last paragraph gives

$$
c = v\big(P_{bcd},\{b,c,d\}\big) = v\big(P_{bcd},\big\{v\big(P_{bcd},\{b,c\}\big), v\big(P_{bcd},\{b,c,d\}\big)\big\}\big) = v\big(P_{bcd},\{b,c\}\big) = b.
$$

[44] It is important not to confuse the agenda $S(A)$ with the agenda $S|_A$. While the former refers to the subgame at node A in S, the latter refers to the pruned agenda on A obtained from S. As Figure 2 illustrates, these agendas may be quite different.

Case (ii). The reasoning is similar to Case (i). Since (B', C') splits D, Claim 4(i) implies $v(P, \{b, c, d\}) = v(P, \{v(P, \{b, d\}), c\})$ and so $v(P_{bcd}, \{b, c, d\}) = c$. Combining this with the formula before Case (i) gives $c = v(P_{bcd}, \{b, c, d\}) = b$. □

Claim 7. *If (B, C) splits $A \in \mathbf{X}$ and (B', C') splits $D \in \mathbf{X}$ for $\varnothing \subset B \cap C \subset D \subseteq C$ and $B \cap C \nsubseteq B'$, then $B \cap C = C'$.*

Proof. Since $B \cap C \nsubseteq B'$, $B \cap C \subseteq C'$ by Claim 6. By way of contradiction, suppose that $B \cap C \subset C'$. Fix alternatives $b \in B' \setminus C'$, $c \in (B \cap C) \setminus B'$, and $d \in C' \setminus (B \cap C)$. Since $B \cap C \subset C'$, $b \in B' \setminus (B \cap C)$. There are two cases: (i) $d \in B' \cap C'$ and (ii) $d \in C' \setminus B'$. Since (B, C) splits A, $v(P, \{b, c, d\}) = v(P, \{c, v(P, \{b, c, d\})\})$ by Claim 4(i). For cases (i) and (ii), this entails a contradiction for $v(P_{bdc}, \{b, c, d\})$. The reasoning is similar to the proof of Claim 6. □

Claim 8. *If (B, C) splits $A \in \mathbf{X}$ and $\varnothing \subset B \cap C \subset D \subseteq C$, then either (i) $(B', B \cap C)$ splits D for some $B' \subset D$ or (ii) (B', C') splits D for some $B', C' \supset B \cap C$.*

Proof. Let (B', C') denote a splitting of D. Given Claims 6 and 7, (i) and (ii) are the only possibilities for (B', C'). □

Lemma 1. *The construction $\mathbb{S}^v$ defines a simple agenda.*

Proof. By Claim 5, the construction in Definition 8 is well defined. Since v satisfies IS, the construction is finite and, thus, defines an agenda $\mathbb{S}^v$. By construction, the agenda $\mathbb{S}^v$ is history-independent. From Claim 8 and Definition 8, it follows that $\mathbb{S}^v$ also satisfies Property $(*)$ (as defined after Remark 5). As such, $\mathbb{S}^v$ is persistent by Remark 5. □

Given Lemma 1, it remains to show that $\mathbb{S}^v$ implements v.

Lemma 2. *The simple agenda $\mathbb{S}^v$ implements v.*

Proof. The proof is by strong induction on $|X| \equiv m$. The base cases $m = 2, 3$ follow from IS and the definition of $\mathbb{S}^v$. To complete the induction, suppose that the claim holds for $m \leq n$ and consider $m = n + 1$.

Given Claim 3 and Remark 1, it suffices to show that $v(P, X) = \text{SVO}[\mathbb{S}^v; P]$ for all $P \in \mathbf{P}$.[45] To show this, let B and C denote the successors of the root X in $\mathbb{S}^v$. Consider the restricted decision rules v_B and v_C that coincide with v on $\mathbf{B}$ and $\mathbf{C}$, respectively. By the induction hypothesis, $\mathbb{S}_B^{v_B}$ implements v_B and $\mathbb{S}_C^{v_C}$ implements v_C. Since $\mathbb{S}^v(B) = \mathbb{S}_B^{v_B}$ and $\mathbb{S}^v(C) = \mathbb{S}_C^{v_C}$ by construction, $v_B(P, B) = SVO[\mathbb{S}^v(B); P]$ and $v_C(P, C) = SVO[\mathbb{S}^v(C); P]$. Since v selects the Condorcet winner between two alternatives (by ILA), "backward induction" requires $SVO[\mathbb{S}^v; P] = v(P, \{v_B(P, B), v_C(P, C)\})$. Then, by definition of v_B and v_C, $SVO[\mathbb{S}^v; P] = v(P, \{v(P, B), v(P, C)\})$. Since (B, C) splits X, $v(P, \{v(P, B), v(P, C)\}) = v(P, X)$. Combining these observations gives $v(P, X) = v(P, \{v(P, B), v(P, C)\}) = SVO[\mathbb{S}^v; P]$. □

[45] To see this, fix some $(P, A) \in \mathbf{P} \times \mathbf{X}$. Since $v(P, A) = v(P^A, X)$ (by Claim 3) and $\text{SVO}[\mathbb{S}^v; P^A] = \text{SVO}[\mathbb{S}^v|_A; P]$ (by Remark 1), showing $v(P^A, X) = \text{SVO}[\mathbb{S}^v; P^A]$ establishes $v(P, A) = v(P^A, X) = \text{SVO}[\mathbb{S}^v; P^A] = \text{SVO}[\mathbb{S}^v|_A; P]$ for all $(P, A) \in \mathbf{P} \times \mathbf{X}$.

A.4.2 *Necessity* The argument given in the main text shows the necessity of ILA. The following claims establish the necessity of IS.

CLAIM 9. *Every history-independent agenda is nonrepetitive.*

PROOF. Fix a history-independent agenda $\mathtt{T}$ and consider a nonterminal node q, where $B \equiv \ell(q)$. By way of contradiction, suppose that $\ell(q_y) = B$ and $\ell(q_n) \equiv C \subseteq B$. By history-independence, the successors of q_y are labeled B and C. Repeating this argument, there is a countably infinite path down the agenda from q where every node is labeled by B. This contradicts the finiteness of $\mathtt{T}$. □

CLAIM 10. *If $\mathtt{S}$ is a simple agenda where B and C are the two successors of the root X, then* $\text{SVO}[\mathtt{S}; P] = x \in B \cap C$ *implies* $\text{SVO}[\mathtt{S}(B); P] = x = \text{SVO}[\mathtt{S}(C); P]$.

PROOF. The proof is by strong induction on $|X| \equiv m$. Since $\mathtt{S}$ is nonrepetitive by Claim 9, the base cases $m = 2, 3$ are trivial. To complete the induction, suppose that the claim holds for $m \leq n$ and consider $m = n + 1$. By way of contradiction, suppose that $\text{SVO}[\mathtt{S}; P] = \text{SVO}[\mathtt{S}(B); P] \equiv x M_P c \equiv \text{SVO}[\mathtt{S}(C); P]$.

First, consider $\mathtt{S}(B)$. Since $\mathtt{S}$ is simple, so is $\mathtt{S}(B)$. By persistence, every path to a terminal node t below B passes through a node B_t^* (potentially B itself) with successors B_t and $B \cap C$. Let $T(B, x) \equiv \{t : x \in B_t\}$. There are two cases: (i) $T(B, x) = \varnothing$ and (ii) $T(B, x) \neq \varnothing$.

Case (i). Since $\text{SVO}[\mathtt{S}(B); P] = x$ and $x \notin B_t$ for all t below B, "backward induction" requires $\text{SVO}[\mathtt{S}(B \cap C); P] = x$.

Case (ii) Since $\text{SVO}[\mathtt{S}(B); P] = x$, "backward induction" requires $\text{SVO}[\mathtt{S}(B_t^*); P] = x$ for some $t \in T(B, x)$. Since $\mathtt{S}(B_t^*)$ is simple and $|B_t^*| \leq n$, the induction hypothesis implies $\text{SVO}[\mathtt{S}(B \cap C); P] = x$.

Next consider $\mathtt{S}(C)$. If $c \in B \cap C$, the same reasoning as in the last paragraph establishes $\text{SVO}[\mathtt{S}(B \cap C); P] = c$. Since this is a contradiction, $c \in C \setminus B$. By persistence, every path to a terminal node t below C passes through a node C_t^* (potentially C itself) with successors C_t and $B \cap C$. Let $T(C, c) \equiv \{t : c \in C_t\}$. By construction, $T(C, c) \neq \varnothing$. Since $\text{SVO}[\mathtt{S}(C); P] = c$, "backward induction" requires $\text{SVO}[\mathtt{S}(C_t); P] = c$ for some $t \in T(C, c)$. Since $\text{SVO}[\mathtt{S}(B \cap C); P] = x$ and $x M_P c$, however, $\text{SVO}[\mathtt{S}(C_t^*); P] = x$. By "backward induction," it follows that $\text{SVO}[\mathtt{S}(C_t); P] \neq c$ which is a contradiction. □

CLAIM 11. *If $\mathtt{S}$ is a simple agenda where B and C are the successors of the root X, then* $\text{SVO}[\mathtt{S}|_B; P] = \text{SVO}[\mathtt{S}(B); P]$.

PROOF. If $B \cap C = \varnothing$, then $\mathtt{S}|_B = \mathtt{S}(B)$ by definition and the claim follows. Suppose that $B \cap C \neq \varnothing$. Fix a profile $P \in \mathbf{P}$ such that $\text{SVO}[\mathtt{S}|_B; P] = b$. By Remark 1, $\text{SVO}[\mathtt{S}; P^B] = b$. Since P and P^B coincide on B, the proof is complete if $\text{SVO}[\mathtt{S}(B); P^B] = b$. (In that case, $\text{SVO}[\mathtt{S}(B); P] = b = \text{SVO}[\mathtt{S}|_B; P]$.) To establish $\text{SVO}[\mathtt{S}(B); P^B] = b$, there are two cases: (i) $b \in B \setminus C$ and (ii) $b \in B \cap C$.

Case (i). Since $b \in B \setminus C$, "backward induction" requires $\text{SVO}[\mathtt{S}(B); P^B] = b$.

Case (ii). Since $\text{SVO}[\mathtt{S}; P^B] = b \in B \cap C$, Claim 10 implies $\text{SVO}[\mathtt{S}(B); P^B] = b$. □

Lemma 3. *If a decision rule v is implementable by simple agenda, then it satisfies IS.*

Proof. Suppose that v is implemented by $\mathtt{S}$. The proof is by strong induction on $|X| \equiv m$. Since $\mathtt{S}$ is nonrepetitive by Claim 9, the base cases $m = 2, 3$ are trivial. To complete the induction, suppose that the claim holds for $m \le n$ and consider $m = n + 1$. Let B and C denote the successors of the root X, and fix an issue A with three or more alternatives. Up to symmetry, there are two possibilities: (a) $A \nsubseteq B, C$ or (b) $A \subseteq B$.

Case (a). In this case, $(B \cap A, C \cap A)$ splits A; that is, it satisfies conditions (i)–(iii) of the definition. (i) Since $B \cup C = X$, $(B \cap A) \cup (C \cap A) = (B \cup C) \cap A = X \cap A = A$. (ii) By contradiction, suppose that $B \cap A = (B \cap A) \cap (C \cap A)$. Then $A = (B \cap A) \cup (C \cap A) = [(B \cap A) \cap (C \cap A)] \cup (C \cap A) = C \cap A$, so $A \subseteq C$ which is a contradiction. (iii) Note that $v(P, B) \equiv \mathrm{SVO}[\mathtt{S}|_B; P] = \mathrm{SVO}[\mathtt{S}(B); P]$ and $v(P, C) \equiv \mathrm{SVO}[\mathtt{S}|_C; P] = \mathrm{SVO}[\mathtt{S}(C); P]$ by Claim 11. Since $v(P, X) \equiv \mathrm{SVO}[\mathtt{S}; P]$ as well, "backward induction" requires $v(P, X) = v(P, \{v(P, B), v(P, C)\})$. Combined with Remark 1, this identity implies

$$\begin{aligned} v(P, A) &= v\big(P^A, X\big) = v\big(P^A, \big\{v\big(P^A, B\big), v\big(P^A, C\big)\big\}\big) = v\big(P^A, \big\{v\big(P^{B\cap A}, X\big), v\big(P^{C\cap A}, X\big)\big\}\big) \\ &= v\big(P^A, \big\{v(P, B \cap A), v(P, C \cap A)\big\}\big) = v\big(P^{\{v(P, B\cap A), v(P, C\cap A)\}}, X\big) \\ &= v\big(P, \big\{v(P, B \cap A), v(P, C \cap A)\big\}\big). \end{aligned}$$

Case (b). By Claim 11, $v(P, B) \equiv \mathrm{SVO}[\mathtt{S}|_B; P] = \mathrm{SVO}[\mathtt{S}(B); P]$. Then, by Remark 1, $\mathtt{S}(B)$ implements the restricted decision rule v_B that coincides with v on $\mathbf{B}$. Since $\mathtt{S}$ is nonrepetitive by Claim 9, $B \subset X$ so that $|B| \le n$. Since $\mathtt{S}$ is simple, so is $\mathtt{S}(B)$. By the induction hypothesis, it then follows that v_B has a splitting for A. Suppose that (B_A, C_A) splits A for v_B. Since $v_B(P, D) \equiv v(P, D)$ for all $D \in \mathbf{B}$, (B_A, C_A) also splits A for v. □

A.4.3 *Uniqueness*

Lemma 4. *If v satisfies IS and ILA, then $\mathtt{S}^v$ is the unique simple agenda that implements v.*

Proof. The proof is by strong induction on $|X| \equiv m$. The base cases $m = 2, 3$ follow from IS. To complete the induction, suppose that the claim holds for $m \le n$ and consider $m = n + 1$.

Fix a simple agenda $\mathtt{S}$ that implements v, and let B and C denote the two successors of the root X in $\mathtt{S}$. First, note that (B, C) is the (unique) splitting of X. To see this, it is enough to check conditions (i)–(iii) in the definition of a splitting. (i) Since $\mathtt{S}$ is an agenda, $B \cup C = X$. (ii) Since $\mathtt{S}$ is nonrepetitive by Claim 9, $B, C \neq B \cap C$ by Claim 1. (iii) By Claim 11, $\mathrm{SVO}[\mathtt{S}|_B; P] = \mathrm{SVO}[\mathtt{S}(B); P]$ and $\mathrm{SVO}[\mathtt{S}|_C; P] = \mathrm{SVO}[\mathtt{S}(C); P]$ for all $P \in \mathbf{P}$. Since $\mathtt{S}$ implements v, $v(P, B) = \mathrm{SVO}[\mathtt{S}|_B; P]$ and $v(P, C) = \mathrm{SVO}[\mathtt{S}|_C; P]$. As a result, $v(P, B) = \mathrm{SVO}[\mathtt{S}(B); P]$ and $v(P, C) = \mathrm{SVO}[\mathtt{S}(C); P]$. Since $v(P, X) = \mathrm{SVO}[\mathtt{S}; P]$ as well, $v(P, X) = v(P, \{v(P, B), v(P, C)\})$.

Next consider the restricted rules v_B and v_C that coincide with v on $\mathbf{B}$ and $\mathbf{C}$, respectively. As shown in the last paragraph, $v_B(P, B) = \mathrm{SVO}[\mathtt{S}(B); P]$ and $v_C(P, C) = \mathrm{SVO}[\mathtt{S}(C); P]$ for all $P \in \mathbf{P}$. By Remark 1 and Claim 3, it follows that $\mathtt{S}(B)$ implements

Profile\Agenda	(a)	(b)	(c)
P_{xbc}	c	b	c
P_{xcb}	b	b	c
Outcomes	Majority loser between b and c	Outcome b for both triples	Outcome c for both triples

TABLE 1. Outcomes implemented by splittings (a)–(c).

v_B and $\mathrm{S}(C)$ implements v_C. Since S is simple, so are $\mathrm{S}(B)$ and $\mathrm{S}(C)$. By the induction hypothesis, $\mathrm{S}_B^{v_B}$ (resp. $\mathrm{S}_C^{v_C}$) is the unique simple agenda that implements v_B (resp. v_C). Accordingly, $\mathrm{S}_B^{v_B} = \mathrm{S}(B)$ and likewise $\mathrm{S}_C^{v_C} = \mathrm{S}(C)$. Since (B, C) is the unique splitting of X (as shown in the last paragraph), the definition of S^v implies $\mathrm{S} = \mathrm{S}^v$. □

LEMMA 5. *If v satisfies IS and ILA, then the structure of S^v is determined by the outcomes of v on Condorcet triples.*

PROOF. Fix an issue A such that $|A| > 2$. For any sub-issue $\{x, b, c\} \subseteq A$, there are three potential splittings where b and c appear in separate sub-issues: (a) $(\{b, x\}, \{c, x\})$, (b) $(b, \{c, x\})$, and (c) $(\{b, x\}, c)$. Each of these implements a different combination of outcomes on the two Condorcet triples as shown in Table 1.

Let (B_A, C_A) denote the unique splitting of A. By construction, there exist alternatives $b_A \in B_A \setminus C_A$ and $c_A \in C_A \setminus B_A$. Then the outcomes of v on each issue $\{x, b_A, c_A\}$ involving an alternative $x \in A$ must coincide with one of the possibilities in Table 1. This observation may be used to pin down alternatives $b_A, c_A \in A$ (perhaps non-uniquely). Again using Table 1, one can then describe the unique splitting (B_A, C_A) of A in terms of Condorcet triples. In particular,

$$B_A \equiv \{b_A\} \cup \big\{x \in A \setminus \{b_A, c_A\} : \text{the outcomes on } \{x, b_A, c_A\} \text{ are type-(a) or type-(c)}\big\},$$

$$C_A \equiv \{c_A\} \cup \big\{x \in A \setminus \{b_A, c_A\} : \text{the outcomes on } \{x, b_A, c_A\} \text{ are type-(a) or type-(b)}\big\}.$$

It follows that the unique splitting of A is determined by the outcomes of v on Condorcet triples. □

A.5 *Proof of Theorem 2*

Note. For each of the Claims 12–14 below, I assume that v satisfies IS and ILA.

CLAIM 12. *If a_* is marginal for $A \in \mathbf{X}$ and $x \in A \setminus a_*$, then a_* is marginal for $A \setminus x$.*

PROOF. Fix some $x \in A \setminus a_*$. By way of contradiction, suppose that $v(P, A \setminus x) = a_*$ for some profile P, where a_* is not the Condorcet winner in $A \setminus x$. By Claim 2, $v(P_x, A \setminus x) = v(P, A \setminus x)$ for any profile $P^{X \setminus x}$ that coincides with P except x is demoted to Condorcet loser in X. Moreover, $v(P^{X\setminus x}, A) = v(P^{X\setminus x}, A \setminus x)$ by ILA. As a result, $v(P^{X\setminus x}, A) = a_*$, which contradicts the assumption that a_* is marginal for A. □

Claim 13. *Every $A \in \mathbf{X}$ has at most two marginal alternatives.*

Proof. Suppose otherwise. Denote any three marginal alternatives in A by x, y, and z, and consider the Condorcet triple P_{xyz} (as defined in the text). Then $v(P_{xyz}, \{x, y, z\}) \notin \{x, y, z\}$ by Claim 12, which is a contradiction. □

Claim 14. *Suppose that (B, C) splits $A \in \mathbf{X}$, where $|A| > 2$ and a_* is marginal for A. Then (i) if $a_* \in C \setminus B$, then $(B, C) = (b, A \setminus b)$ for some $b \neq a_*$, and (ii) if $a_* \in B$ and a_{**} is also marginal for A, then $a_{**} \in B$.*

Proof. (i) By way of contradiction, suppose that $|B \setminus C| \geq 2$. Fix $b, b' \in B \setminus C$ and consider the triple $P_{a_*bb'}$. By Claim 4, the splitting of $\{a_*, b, b'\}$ is $(\{b, b'\}, a_*)$, and $v(P_{a_*bb'}, \{a_*, b, b'\}) = v(P_{a_*bb'}, \{v(P_{a_*bb'}, \{b, b'\}), a_*\}) = a_*$. By Claim 12, this contradicts the assumption that a_* is marginal for A. (ii) By way of contradiction, suppose that $a_{**} \in C \setminus B$. Without loss of generality, $|A| > 2$ implies there is some alternative $x \in A$ such that: $x \in B \setminus C$ if $a_* \in B \cap C$ or $x \in B$ if $a_* \in B \setminus C$. In either case, a contradiction obtains along the same lines as (i). □

Claim 15. *Suppose that v satisfies IS and ILA. (i) If each $A \in \mathbf{X}$ such that $|A| > 2$ has two marginal alternatives, then $\mathtt{S}^v$ is Euro–Latin. (ii) If each $A \in \mathbf{X}$ such that $|A| > 2$ has one marginal alternative, then $\mathtt{S}^v$ is Anglo–American.*

Proof. Consider the splitting (B, C) of X and let x_* denote an alternative that is marginal on X.

(i) By Claim 14, the potential splittings of X are (a) $(B, C) = (b_1, X \setminus b_1)$ for $x_* \neq b_1$ and (b) $x_* \in B \cap C$ with $b \in B \setminus C$ and $c \in C \setminus B$. For (b), Claim 4(ii) implies that $\{x_*, b, c\}$ has one marginal alternative x_*, which is a contradiction. It follows that the splitting must be (a). Continuing in the same vein on $X \setminus b_1$ shows that $\mathtt{S}^v$ is Euro–Latin.

(ii) If $|C \setminus B| \geq 1$ (with $b \in B \setminus C$ and $c, c' \in C \setminus B$), then Claim 4(ii) implies that c and c' are marginal on $\{b, c, c'\}$, which is a contradiction. Since $|B \setminus C| \geq 1$ yields a similar contradiction, $(B, C) = (X \setminus c_1, X \setminus b_1)$ for some $b_1 \in B \setminus x_*$ and $c_1 \in C \setminus x_*$ by Claim 14(i). Continuing in the same vein on $X \setminus c_1$ and $X \setminus b_1$ shows that $\mathtt{S}^v$ is Anglo–American. □

Proof of Theorem 2. (Sufficiency). By Claim 15 and Theorem 1. (Necessity) It is clear that Euro–Latin and Anglo–American agendas are simple. By Theorem 1, a decision rule implemented by an agenda of either format must satisfy IS and ILA. Finally, for all issues A such that $|A| > 2$, observe that (a) every rule implemented by Euro–Latin agenda marginalizes the last two alternatives in the priority ordering (which are symmetric) and (b) every rule implemented by Anglo–American agenda marginalizes the last alternative in the priority ordering (see Moulin 1988, p. 250). □

A.6 *Proof of Theorem 3*

Claim 16. *If q is a nonterminal node of a priority agenda $\mathtt{Pr}$ such that $|\ell(q)| > 2$, then $\ell(q_y) = \ell(q) \setminus d$ or $\ell(q_n) = \ell(q) \setminus d$, where d is one of the two highest priority alternatives at q.*

Proof. Label the alternatives $\ell(q) \equiv \{d_1, \dots, d_k\}$ according to a priority ordering $\succ$ for $\texttt{Pr}$. If $d_1, d_2 \in \ell(q_y)$, then $\ell(q_y)$ must also contain every alternative with lower priority. As a result, $\ell(q_y) = \ell(q)$, which contradicts the fact that $\texttt{Pr}$ is nonrepetitive. (Since it is a priority agenda, $\texttt{Pr}$ must be history-independent; by Claim 9, it must also be nonrepetitive.) Without loss of generality, suppose $d_1 \in \ell(q_y) \setminus \ell(q_n)$. If $d_3 \in \ell(q_y)$, then $\ell(q_y) = \ell(q) \setminus d_2$ by definition of priority agendas. Similarly, $\ell(q_n) = \ell(q) \setminus d_1$ if $d_3 \in \ell(q_n)$. □

Given a priority agenda $\texttt{Pr}_A$, let $\succ^p_A$ denote any ordering on A that satisfies the following requirement: for all $a \in A$, there is a node in $\texttt{Pr}_A$ labeled $\{d \in A : d \not\succ^p_A a\}$. By Claim 16, there is some such ordering for each priority agenda.

Remark 6. If $\texttt{Pr}_A$ is a priority agenda, then every node takes the form in Figure 3 for $\succ^p_A$.

Proof. The proof is by strong induction on $|A| \equiv m$. The base cases $m = 2, 3$ are straightforward. To complete the induction, suppose that the claim holds for $m \leq n$ and consider $m = n + 1$. Label the alternatives in $A \equiv \{a_1, \dots, a_m\}$ according to $\succ^p_A$. Then the successors of A are $B \equiv \{a_1, a_i, \dots, a_m\}$ and $C \equiv \{a_2, \dots, a_m\}$. Accordingly, the root node takes the desired form for $\succ^p_A$. Since $\texttt{T}_A(B)$ and $\texttt{T}_A(C)$ are priority agendas, the induction hypothesis implies that every node starting from B (resp. C) takes the desired form for $\succ^p_B$ (resp. $\succ^p_C$).

Since $\texttt{Pr}_A$ is nonrepetitive and persistent, one successor of B must contain a_1 while the other must be $\{a_i, \dots, a_m\}$. Then $\succ^p_B$ may be taken to coincide with $\succ^p_A$ on B (by defining $a_1 \succ^p_B b$ for all $b \in \{a_i, \dots, a_m\}$ and letting $\succ^p_B$ coincide with $\succ^p_A$ on $B \setminus a_1$). By construction, $\succ^p_C$ is the restriction of $\succ^p_A$ to C. □

Claim 17. *If v satisfies IS, ILA, and mandatory marginalization, then $\texttt{S}^v$ is a priority agenda.*

Proof. By Theorem 1, $\texttt{S}^v$ is simple. I show that there is some priority ordering $\succ$ for $\texttt{S}^v$. The proof is by induction on $|A| \equiv m$. The base cases $m = 2, 3$ are straightforward.

To complete the induction, suppose that the claim holds for $m \leq n$ and consider $m = n + 1$. By mandatory marginalization, there is some marginal alternative x_* on X. Consider any issue $A \in \mathbf{X}$ such that $|A| > 2$ and $x_* \in A$. By Claim 12, x_* is also marginal on A. Let (B, C) denote the splitting of A. By Claims 4 and 14, $(B \setminus x_*, C \setminus x_*)$ splits A. It follows that $\texttt{S}^v|_{X \setminus x_*} = \texttt{S}^v_{X \setminus x_*}$ (i.e., the restriction of $\texttt{S}^v$ to $X \setminus x_*$ coincides with the agenda on $X \setminus x_*$ built using Definition 8).

By the induction hypothesis, $\texttt{S}^v_{X \setminus x_*}$ is a priority agenda. Let $\succ^*$ denote a priority ordering for $\texttt{S}^v_{X \setminus x_*}$ and let $\succ$ denote the extension to X where $x \succ x_*$ for all $x \in X \setminus x_*$. I show that $\succ$ is a priority ordering for $\texttt{S}^v$. Clearly, $\succ$ has the desired properties for any nonterminal node A such that $x_* \notin A$ or $|A| = 2$.[46] Next, consider any node A such that $x_* \in A$ and $|A| > 2$. By Claim 14, the splitting of A must be $(b, A \setminus b)$ for some $b \neq x_*$ or

[46]In fact, one can show that there are no such nodes where $x_* \notin A$ and $|A| > 2$.

(B, C) with $|B|, |C| \geq 2$ and $x_* \in B \cap C$. The corresponding node $A \setminus x_*$ in $\mathtt{S}^v_{X \setminus x_*}$ has the splitting $(b, A \setminus \{b, x_*\})$ or $(B \setminus x_*, C \setminus x_*)$. Since $\succ^*$ has the desired properties for $A \setminus x_*$ in $\mathtt{S}^v|_{X \setminus x_*}$, it follows that $\succ$ has the desired properties for A in $\mathtt{S}^v$. □

Proof of Theorem 3. (Sufficiency). By Claim 17 and Theorem 1. (Necessity) Suppose that v is implemented by a priority agenda $\mathtt{Pr}$ where the alternatives of $X \equiv \{x_1, \dots, x_m\}$ are indexed by the priority ordering $\succ$. Since $\mathtt{Pr}$ is simple by definition, Theorem 1 implies that v satisfies IS and ILA.

To show that v satisfies mandatory marginalization, it suffices to show that x_m is marginal for X.[47] The proof is by strong induction on m. The base cases $m = 2, 3$ are straightforward. To complete the induction, suppose that the claim holds for $m \leq n$ and consider $m = n + 1$. Without loss of generality, the successors of X are $B \equiv \{x_1, x_i, \dots, x_m\}$ and $C \equiv \{x_2, \dots, x_m\}$. Then $\mathtt{Pr}(B)$ and $\mathtt{Pr}(C)$ are priority agendas (with priorities that restrict $\succ$ to B and C). By Claim 10, $\text{SVO}[\mathtt{Pr}; P] = x_m$ only if $\text{SVO}[\mathtt{Pr}(B); P] = x_m = \text{SVO}[\mathtt{Pr}(C); P]$. Since x_m is the lowest priority alternative in B and C, the induction hypothesis implies that $\text{SVO}[\mathtt{Pr}(B); P] = x_m = \text{SVO}[\mathtt{Pr}(C); P]$ only if x_m is the Condorcet winner on (P, B) and (P, C). As such, x_m must also be the Condorcet winner on (P, X). □

A.7 *Proof of Theorem 4*

Note. For each of the Claims 18–20 below, I assume that v satisfies IS and ILA.

Claim 18. *The following statements are equivalent: (a) v satisfies directed marginalization and (b) for all $A \in \mathbf{X}$ such that $|A| = 4$, no $x_* \in A$ is uniquely marginal for all $D \subset A$ such that $|D| = 3$ and $x_* \in D$.*

Note: For ease of presentation in the sequel, I refer to the property specified in (b) above as Property $(**)$.

Proof. (a) ⇒ (b) Toward a contradiction, suppose that $A \equiv \{x_*, y, z, w\}$ violates Property $(**)$ with respect to $x_* \in A$. Since v_A (i.e., the restriction of v to $\mathbf{A}$) satisfies IS and ILA, Theorem 1 implies that v_A is implemented by simple agenda. For any pair of alternatives $a, b \in \{y, z, w\}$, the splitting of $\{x_*, a, b\}$ is then $(\{x_*, a\}, \{x_*, b\})$. Up to symmetry, this leaves two possibilities for the splitting of A: (i) $(A \setminus z, A \setminus y)$ and (ii) $(\{x_*, y\}, A \setminus y)$.

Case (i). In this case, IS implies

$$v(P, A) = v\big(P, \big\{v\big(P, \big\{v\big(P, \{x_*, y\}\big), v\big(P, \{x_*, w\}\big)\big\}\big), v\big(P, \big\{v\big(P, \{x_*, z\}\big), v\big(P, \{x_*, w\}\big)\big\}\big)\big\}\big),$$

As a result, x_* is marginal for A. Since none of the alternatives y, z, w is prioritized, this contradicts directed marginalization.

[47] By Remark 1, it follows that x_m is marginal for any issue $A \subseteq X$ such that $x_m \in A$. Since the pruned agenda $\mathtt{Pr}|_{X'}$ on $X' \equiv \{x_1, \dots, x_{m-1}\}$ is a priority agenda (with a priority that restricts $\succ$ to X'), the same argument shows that x_{m-1} is marginal for any issue $A \subseteq X'$ such that $x_{m-1} \in A$. By progressively deleting the highest index alternative among those remaining and repeating this argument, it follows that there exists a marginal alternative for all A such that $|A| > 2$.

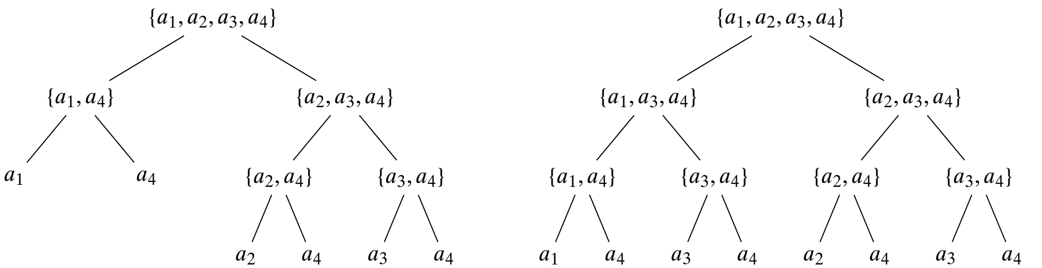
(ii) In this case, IS implies

$$v(P,A) = v(P, \{v\big(P, \big\{v\big(P, \{x_*, y\}\big), v\big(P, \big\{v\big(P, \{x_*, z\}\big), v\big(P, \{x_*, w\}\big)\big\}\big)\big\}\big).$$

As in the previous case, x_* is marginal for A, but none of y, z, w is prioritized, which contradicts directed marginalization.

(b) $\Rightarrow$ (a) It suffices to verify that v does not violate directed marginalization for any issue A such that $|A| = 4$. To see this, fix some issue B such that $|B| > 4$ with a marginal alternative $x_* \in B$. By Claim 12, x_* is marginal for every issue $x_* \in A \subset B$ such that $|A| = 4$. Suppose that $v(P,A) = a^*$ for some profile P such that $x_* M_P a^*$ (i.e., v prioritizes a^* for A). Then $v(P^A, B) = a^*$ by Claim 3. What is more, $x_* M_{P^A} a^*$, so v prioritizes a^* for B.

To complete the proof, fix an issue A such that $|A| = 4$ with a marginal alternative $x_* \in A$. By way of contradiction, suppose that v does not prioritize any alternative for A. Since v_A (i.e., the restriction of v to $\mathbf{A}$) satisfies IS, ILA, and mandatory marginalization, Theorem 3 implies that v_A is implemented by a priority agenda. Labeling the alternatives $A \equiv \{a_1, \ldots, a_4\}$ (in terms of priority), there are six possible priority agendas on A. The only two that prioritize nothing for A are



In either case, a_4 is uniquely marginal on $\{a_i, a_j, a_4\}$ for all distinct $i, j \in \{1, 2, 3\}$, so v violates Property $(**)$ on A. $\square$

Claim 19. *Suppose that v satisfies directed marginalization. If (B, C) splits $A \in \mathbf{X}$ and $B \cap C \neq \varnothing$, then $(B', B \cap C)$ splits B and $(B \cap C, C')$ splits C for issues $B' \subset B$ and $C' \subset C$ such that $B' \cap C' = \varnothing$.*

Proof. The proof is by strong induction on $|B \cap C| \equiv m$. For the base case $m = 1$, the claim is trivial if $|A| = 3$. Suppose $|A| \geq 4$. Let (B_C, C_C) denote the splitting of C and define $B \cap C \equiv \{x\}$. By way of contradiction, suppose that $B_C, C_C \neq \{x\}$. By Claim 8, $x \in B_C \cap C_C$. Since $|A| \geq 4$, there are $b' \in B_C \setminus (B \cup C_C)$ and $c' \in C_C \setminus (B \cup B_C)$. Since (B, C) splits X, there is some $b \in B \setminus C$. Since $x \in B, B_C, C_C$, it follows that x violates Property $(**)$ on $A \equiv \{x, b, b', c'\}$. By Claim 18, this contradicts directed marginalization, so the splitting of C is $(x, C \setminus x)$. Since the same reasoning shows that the splitting of B is $(B \setminus x, x)$, this establishes the base case.

To complete the induction, suppose that the claim holds for $m \leq n$ and consider $m = n+1$. Fix some $x \in B \cap C$ and set $Y \equiv X \setminus x$. By Claim 4(ii), the splitting of Y is $(B \setminus x, C \setminus x)$. By the induction hypothesis, the splittings of $B \setminus x$ and $C \setminus x$ are $(B', (B \cap C) \setminus x)$ and

$((B \cap C) \setminus x, C')$ with $B' \cap C' = \varnothing$. By Claims 4(i) and 6, it follows that the splitting of C is $(B \cap C, C'')$ with $C'' = C'$ or $C'' = C' \cup x$. By way of contradiction, suppose that $C'' = C' \cup x$. Fix alternatives $b \in B \setminus C$, $b' \in (B \cap C) \setminus C''$ and $c' \in C'' \setminus B$. By the same reasoning as in the base case, x violates Property $(**)$ on $A \equiv \{x, b, b', c'\}$, so the splitting of C is $(B \cap C, C')$. Similarly, the splitting of B is $(B', B \cap C)$. □

Claim 20. *If v satisfies directed marginalization, then $\mathtt{S}^v$ is convex.*

Proof. The proof is by strong induction on $|X| \equiv m$. The base cases $m = 2, 3$ are trivial. For $m = 4$, directed marginalization rules out only the two agendas in Claim 18. The seven remaining simple agendas on four alternatives are convex.

To complete the induction, suppose that the claim holds for $m \leq n$ and consider $m = n + 1$. Let B and C denote the successors of X. By the induction hypothesis, there are convex orderings $\succ_B$ and $\succ_C$ for $\mathtt{S}^v(B)$ and $\mathtt{S}^v(C)$, respectively. Given Claim 19, it suffices to show that $\succ_B$ and $\succ_C$ may be chosen so that they agree on $B \cap C$.

First, consider the splitting $(B \cap C, C')$ of C and suppose that $B \cap C' \neq \varnothing$. (Otherwise, no conflict between $\succ_B$ and $\succ_C$ need arise; in particular, there is a convex ordering $\succ'_C$ on C for $\mathtt{S}^v(C)$ that agrees with $\succ_B$ on $B \cap C$ and $\succ_C$ on C'.) By Claim 19, it follows that the splitting of $B \cap C$ is $(B \cap C', C'')$. Likewise, suppose $B' \cap C \neq \varnothing$ for the splitting $(B', B \cap C)$ of B. Then the splitting of $B \cap C$ is $(B'', B' \cap C)$. Combining these observations implies that the splitting of $B \cap C$ is $(B' \cap C, B \cap C')$. What is more, $(B' \cap C) \cap (B \cap C') = \varnothing$ by Claim 19.

Up to right–left symmetry, there are two possibilities for any pair $x, y \in B \cap C$: (i) $x \in B' \cap C$ and $y \in B \cap C'$ or (ii) $x, y \in B' \cap C$.

Case (i). Fix alternatives $b \in B \setminus C$ and $c \in C \setminus B$. Up to right–left symmetry, the convexity of $\mathtt{S}^v(B)$ and $\mathtt{S}^v(C)$ implies $b \succ_B x \succ_B y$ and $x \succ_C y \succ_C c$. It follows that $\succ_B$ and $\succ_C$ need not disagree on the ranking of $x, y \in B \cap C$.

Case (ii). As in Case (i), there need be no conflict between $\succ_B$ and $\succ_C$. In particular, there is a convex ordering $\succ'_C$ for $\mathtt{S}^v(C)$ that agrees with $\succ_B$ on $B' \cap C$ and agrees with $\succ_C$ on $C \setminus B' = C'$. □

Proof of Theorem 4. (Sufficiency). By Claim 20 and Theorem 1. (Necessity) Suppose that v is implemented by a convex agenda $\mathtt{Conv}$ with ordering $\succ$. Since $\mathtt{Conv}$ is simple, Theorem 1 implies that v satisfies IS and ILA.

To show that v satisfies directed marginalization, it suffices to show that it satisfies Property $(**)$ by Claim 18. Fix an issue $A \equiv \{a_1, a_2, a_3, a_4\}$ indexed by $\succ$. By definition of convex agendas, there is some node $A' \supseteq A$ with successors B' and C' such that $a_1 \in B' \setminus C'$ and $a_4 \in C' \setminus B'$. By Claim 4(ii), the splitting of A must respect $\succ$ in the sense of Figure 4. Up to left–right symmetry, there are four possibilities: (i) $(\{a_1, a_2, a_3\}, \{a_4\})$, (ii) $(\{a_1, a_2\}, \{a_3, a_4\})$, (iii) $(\{a_1, a_2, a_3\}, \{a_3, a_4\})$, and (iv) $(\{a_1, a_2, a_3\}, \{a_2, a_3, a_4\})$. For cases (i) and (ii), Claim 4(ii) implies that no alternative in A violates Property $(**)$. For cases (iii) and (iv), it suffices to check that a_3 does not violate Property $(**)$. (The other alternatives are covered by Claim 4(ii) and/or symmetry.) For case (iii), Claim 4(i) requires the splitting of $\{a_1, a_2, a_3\}$ to be $(\{a_1, a_2\}, \{a_3\})$. As such, a_3 is not even marginal for $\{a_1, a_2, a_3\}$. For case (iv), Claim 4(i) requires the splitting of $\{a_1, a_2, a_3\}$ to be $(\{a_1\}, \{a_2, a_3\})$ or $(\{a_1, a_2\}, \{a_2, a_3\})$. In either case, a_3 is not uniquely marginal for $\{a_1, a_2, a_3\}$. □

A.8 *Proof of Theorem 5 and Remark 4*

Proof of Theorem 5. Suppose that the decision rule v is implemented by a simple agenda $\mathtt{S}$. It suffices to show that v is monotonic at X. (Since $v(P,A)=v(P^A,X)$ by Remark 1, the result follows for all $A\in\mathbf{X}$.) The proof is by strong induction on $|X|\equiv m$. The base cases $m=2,3$ are trivial. To complete the induction, suppose that the claim holds for $m\le n$ and consider $m=n+1$. Let B and C denote the successors of the root X in $\mathtt{S}$. For a profile P such that $v(P,X)=\mathrm{SVO}[\mathtt{S};P]=x$, there are two cases: (i) $x\in B\cap C$ and (ii) $x\in B\setminus C$.

Case (i). By Claim 10, $\mathrm{SVO}[\mathtt{S}(B);P]=\mathrm{SVO}[\mathtt{S}(C);P]=x$. Since $\mathtt{S}$ is nonrepetitive by Claim 9, $B,C\subset X$ so that $|B|,|C|\le n$. Since $\mathtt{S}$ is simple, so are $\mathtt{S}(B)$ and $\mathtt{S}(C)$. By the induction hypothesis, it follows that $\mathrm{SVO}[\mathtt{S}(B);P^{\uparrow x}]=x=\mathrm{SVO}[\mathtt{S}(C);P^{\uparrow x}]$. Then "backward induction" requires $v(P^{\uparrow x},X)=\mathrm{SVO}[\mathtt{S};P^{\uparrow x}]=x$.

Case (ii). By the same reasoning as Case (i), $\mathrm{SVO}[\mathtt{S}(B);P^{\uparrow x}]=\mathrm{SVO}[\mathtt{S}(B);P]=x$. Moreover, $\mathrm{SVO}[\mathtt{S}(C);P^{\uparrow x}]=\mathrm{SVO}[\mathtt{S}(C);P]$ since $x\notin C$. Then "backward induction" requires $v(P^{\uparrow x},X)=\mathrm{SVO}[\mathtt{S};P^{\uparrow x}]=x$. □

Proof of Remark 4. First, observe that the agendas $\mathtt{A}'\equiv\mathtt{A}(\{ba',s,sa'',\varnothing\})$ and $\mathtt{A}''\equiv\mathtt{A}(\{b,s,sa'',\varnothing\})$ below the root node of the agenda $\mathtt{A}$ in Example 7 are simple. As a result, $\mathtt{A}'$ and $\mathtt{A}''$ satisfy preference monotonicity by Theorem 5. Letting $\mathrm{SVO}[\mathtt{A};P]\equiv x$, there are three cases: (a) $x\in\{b,ba'\}$, (b) $x\in\{s,sa''\}$, and (c) $x=\varnothing$.

(a) Let $x=b$. Since b appears only in $\mathtt{A}''$, "backward induction" requires $\mathrm{SVO}[\mathtt{A}'';P]=b$. By preference monotonicity of $\mathtt{A}''$, $\mathrm{SVO}[\mathtt{A}'';P^{\uparrow b}]=b$. Since $\mathrm{SVO}[\mathtt{A}';P]=\mathrm{SVO}[\mathtt{A}';P^{\uparrow b}]$ as well, "backward induction" implies $\mathrm{SVO}[\mathtt{A};P^{\uparrow b}]=b$.

(b) Let $x=s$. Since s appears in both sub-agendas at the root, there are two cases:

  (i) If $\mathrm{SVO}[\mathtt{A}';P]=s=\mathrm{SVO}[\mathtt{A}'';P]$, then $\mathrm{SVO}[\mathtt{A}';P^{\uparrow s}]=s=\mathrm{SVO}[\mathtt{A}'';P^{\uparrow s}]$ by preference monotonicity of $\mathtt{A}'$ and $\mathtt{A}''$. By "backward induction," $\mathrm{SVO}[\mathtt{A};P^{\uparrow s}]=s$.

  (ii) If $\mathrm{SVO}[\mathtt{A}';P]\neq s=\mathrm{SVO}[\mathtt{A}'';P]$, then $\mathrm{SVO}[\mathtt{A}''(\{b,s,\varnothing\});P]=s$ so that $sM_P\varnothing$. Since $\mathtt{A}'$ is simple, $\mathrm{SVO}[\mathtt{A}';P]=ba'$ (resp. $\varnothing$) implies $\mathrm{SVO}[\mathtt{A}'(\{ba',s,\varnothing\});P]=ba'$ (resp. $\varnothing$) by Claim 10, so $\mathrm{SVO}[\mathtt{A}';P]=sa''$. (If $\mathrm{SVO}[\mathtt{A}';P]=ba'$, then $\mathrm{SVO}[\mathtt{A}'(\{ba',s,\varnothing\});P]=ba'$ so that $ba'M_Ps$, which contradicts $\mathrm{SVO}[\mathtt{A};P]=s$; if $\mathrm{SVO}[\mathtt{A}';P]=\varnothing$, then $\varnothing M_Ps$, which is a contradiction.) Since $\mathrm{SVO}[\mathtt{A}';P]=sa''$, $\mathrm{SVO}[\mathtt{A}'(\{ba',sa'',\varnothing\});P]=sa''$. This implies $\mathrm{SVO}[\mathtt{A}'(\{ba',s,\varnothing\});P]=ba'$ (since sM_Psa'' and $\varnothing$) so that $ba'M_Ps$ and $ba'M_P\varnothing$. As a result, $\mathrm{SVO}[\mathtt{A}'(\{ba',s,\varnothing\});P^{\uparrow s}]=s$ (since $sM_P\varnothing$) and $\mathrm{SVO}[\mathtt{A}';P^{\uparrow s}]=s$ (since $\mathrm{SVO}[\mathtt{A}'(\{ba',sa'',\varnothing\});P^{\uparrow s}]=sa''$ and sM_Psa''). Since $\mathrm{SVO}[\mathtt{A}'';P^{\uparrow s}]=s$ by preference monotonicity of $\mathtt{A}'$, $\mathrm{SVO}[\mathtt{A};P^{\uparrow s}]=s$.

(c) Since $\varnothing$ is available in every terminal sub-game, it is the Condorcet winner on P. As a result, $\mathrm{SVO}[\mathtt{A};P^{\uparrow\varnothing}]=\varnothing$.

□

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